DOCUMENT RESUME

ED 059 861 SE 005 876

TITLE	Central Iowa Low Achiever Mathematics Project -- Area Measurement; Graphing Pictures; An Introduction to Flow Charting; First Probability Program.
INSTITUTION	Central Iowa Low-Achiever Mathematics Project, Des Moines.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	69
NOTE	158p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	Arithmetic; Grade 7; Grade 8; Grade 9; Graphs; Instruction; *Instructional Materials; *Low Achievers; *Mathematics; Measurement; Probability; *Secondary School Mathematics

ABSTRACT

The materials in these units are designed especially for the low achiever student in junior high school mathematics. These materials are intended to be a source of new ideas for teachers who are trying to encourage interest, enthusiasm, and participation from low achieving students in mathematics. The four units in this collection contain mathematical materials involving area measurement, graphing, probability, and an introduction to flow charting. This work was prepared under an ESEA Title III contract. (RP)

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Central Iowa Low Achiever Mathematics Project

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SE 005 876

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AREA MEASUREMENT

CENTRAL IOWA LOW-ACHIEVER MATHEMATICS PROJECT
(CILAMP)
1164 - 26th Street
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AREA MEASUREMENT

TEACHER'S AID

(If all else fails, read this.)

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A BRIEF NOTE

The booklet on measurement makes no claims on rigor. The purpose is very simply to acquaint the student with a few of the more familiar figures he may someday be asked to know.

On the mathematical side, the booklet does attempt to make one concept very clear, namely that whenever possible mathematics uses what is already known to advance to the next notion.

Those who are "purists" will very likely find the approach here pretty much intolerable, particularly the slaughter of the circle. Those who can abandon rigor without blushing will hopefully find merit in this presentation.

COMMENTS ON STUDENT PREPARATION

Before attempting this unit the student should have a fairly good grasp of the basic operations with fractions and decimals. In addition, he should be acquainted with measurement of length, and it would be more pleasant if he had at least a passing acquaintance with π, nothing more subtle than knowing that the circumference of a circle is π times the diameter. (For purposes of this booklet it is acceptable, perhaps advisable, to assign π the value 22/7.)

GENERAL COMMENTS ON METHOD

The booklet was written with a particular student in mind, the student whose experience with mathematics has fallen short of pleasure. Assuming that the booklet is being used with these students, reading assignments should not be made. In the first place, the vocabulary was not chosen for reading levels common to many students with a prolonged history of mathematics rejection. Secondly, assigning reading to these students is often equivalant to ommitting the reading. The written sections should be read in class.

If there is reason to believe that student reaction to seeing many pages at once would be adverse, the materials may be handed out a "part" at a time or sometimes a page at a time.

The questions marked "CLASS DISCUSSION" are in no circumstances meant to be assigned to individuals. In most cases their intention is to create student involvement by which, hopefully, small points about a concept may be cleared by the students for themselves.

As is true of all textbooks, the teacher should not feel that the procedure is binding. Convinced that improvements can be made or that procedures can be substituted, a teacher should joyfully reject the text.

STUDENT EQUIPMENT

The equipment needs of each student will be determined largely by which parts of the booklet are ommitted, which parts are done as demonstration, and by which parts are declared to be self-evident truths. No matter what is done, it will be required that each student have a ruler (a foot marked by inches) at his disposal. If the booklet is taken seriously in its entirety, the following equipment will be needed for each student:

1. A ruler
2. 20 squares, each exactly 1 inch by 1 inch
3. 10 rectangles, each exactly 1 inch by 2 inches

4. 12 right triangles, sides forming the right angle measuring 1 inch and 2 inches
5. 5 circles, each $1\frac{1}{4}$ inches in diameter
6. A one-inch ruler (a rectangle 1 inch by $\frac{1}{2}$ inch)

Risking the threats of paper airplanes and spit-balls, it may become desirable to have the students perform the demonstrations outlined by the booklet. In this case a pair of scissors should also be included.

Protractors and compasses are optional. Unless the students have a fairly sound background with these instruments, it is advised that they might do more to cloud issues than to help.

COMMENTS ON INDIVIDUAL "PARTS"

pages 1 - 9

Class level will make one of three approaches appropriate:

(a) Have the students perform all exercises as given.

(b) Demonstrate the concepts for the class.

(c) Briefly summarize for the class.

There are two purposes here: to introduce what goes on in finding area, and to make the point that the unit we choose for finding area is one of convenience, not necessity.

pages 10 - 18

Once again, class level will determine one of the three approaches mentioned above. Choosing one approach for one part should not be binding for another part.

Two concepts will be introduced: We can find area of a rectangle by measuring length, and the area of a rectangle turns out to be length times width. In addition the importance of specifying units is brought out.

It would be advisable to work an example or two before assigning problems since very few examples are included in the student's booklet.

pages 19 - 27

If paper cutting is to be tried, this part most likely lends itself best to the effort. If bewilderment persists following the presentation on pages 23 and 24, several demonstrations might be helpful, using very tall and thin parallelograms.

It might also be advisable to emphasize the fact that knowledge about rectangles helped to conquer a worse figure.

Once again, if there is any suspicion that some students may not be ready for the exercises after the reading, present several examples before assigning problems.

pages 28 - 31

The procedure is different from that in pages 19 - 27. If the students seem confident about pages 19 - 27, it is possible that the area of a triangle could be given as a project without giving the students PART D. PART D of the booklet could then be handed to them later as a summary of what was done.

The comment about examples before assignments prevails.

pages 32 - 34

The procedure is identical to that in pages 28 - 31. If it is desirable to give a project without first seeing the book, this is the best spot. PART E employs exactly the same "constructing a twin" method as was presented in PART D. Allowing the student a victory at this point would be in order.

Once again, examples before assignments.

pages 35-39

PART F affords milestones in "mathematical slushiness." At the same time, falling back on "the end justifies the means," we believe that the abuses are worthy ones. The length of the figure appearing on the top of page 38 is πr only if one follows the not-so-straight line running along the bottom of the figure; and the height is "r" only at selected points in the figure. If the students appear to be skeptics, cutting 16 "pieces of pie" instead of just 8 will result in amazingly close results. In addition, this will suggest the concept of "limit," the only justification for the procedure of PART F.

Statement number 2 appearing at the bottom of page 37 may move too fast for the student. If looks of bewilderment occur on the first reading of the statement, it is strongly suggested that the reading stop, and remain at a halt until every student is confident that, following the not-so-straight base, the length is indeed πr.

For the last time, with a hint of apology, remember examples before assigning exercises.

COMMENTS AND ANSWERS TO EXERCISES

(by page and number of exercise)

2-(a)	4 units
2-(b)	9 units
3-(c)	3 units
4-(a)	8 units
4-(b)	6 units
4-(c)	12 units
5-(a)	8 units
5-(b)	4 units
6-(c)	12 units
7-(a)	1 unit
7-(b)	too difficult
7-(c)	too difficult
8-(a)	too difficult at this point
8-(b)	too difficult at this point
9-(a)	10 units
9-(b)	9 units
9-(c)	11 units
10-(a)	18, 3, 6
11-(b)	21, 3, 7, no, move a square or imagine a square
12-(a)	6, 1, 6
12-(b)	10, 2, 5
14	measure lengths

18-1	80 square inches
18-2	9 square miles
18-3	5/2 square inches
18-4	15 yards
18-5	6160 square feet
18-6	2 square yards (or 18 square feet
18-7	90,000 square yards
24	no
26-1-(a)	2 square inches
26-1-(b)	4 square inches
26-1-(c)	1 square inch
26-1-(d)	1 square inch
26-1-(e)	39/4 sq. inches
27-2	245 square cm.
27-3	18 square fathoms
27-4	312 square inches (or) 13/6 square feet
27-5	9 yards
27-6	50 yards
27-7	50 square feet, 0 square feet
27-8	no

30-1-(a)	2 square inches
30-1-(b)	3/2 square inches
30-1-(c)	1 square inch
30-1-(d)	3 square inches
30-1-(e)	15/4 square inches
31-2	6 square inches
31-3	15/2 square feet
31-4	10 feet
31-5	infinitely many
31-6	there are no "correct" answers - students should look for right triangles and realize that once a base is selected, the height is also determined
31-7	400 square miles
34-1	40 square inches
34-2	49/4 square feet
34-3	7/2 square miles
34-4	30 square inches
34-5	7 yards
34-6	0 links
34-7	no, not if you are willing to call a triangle a trapezoid with one of its bases equal to "0"
39-1	352/7 square cm.
39-2	2662/7 square yards
39-3	divide "c" by 2π to get "r"; then evaluate πr^2
39-4	198/4 square feet
39-5	28 inches
39-6	96/7 square inches
40-1	8 square inches
40-2	6 square inches
40-3	5/2 square inches
40-4	4 square inches
41-5	22/7 square inches
41-6	11 square inches
41-7	8 square inches
41-8	5 square inches
42-9	14 square inches
42-10	66/7 square inches
43-11	6 square thumbs
43-12	286/7 square yards
43-13	45 furlongs
43-14	(a), (c)
43-15	(b), (d)
43-16	8 square feet
43-17	22 feet

AREA MEASUREMENT

STUDENT'S BOOK

By

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PART A

Choosing a unit of measurement

We want to measure area. As we did with length, we must choose a common unit of measure. Although some choices for the unit will make measurement of area easier than other choices, it is also important that everyone agrees to use the same unit. If they didn't, "3 units" could mean different things to different people. (Imagine for a moment what would happen if everyone suddenly decided to make his own ruler exactly as long as he pleased - the shortest member in your class might claim that he was ten feet tall by his ruler, and another member of the class might insist that the football field was only forty-six feet long by his ruler.)

With the unit we choose, a little piece of area, we will cover the area we want to measure. The rules for covering the area we want to measure are similar to the rules we use for measuring length:

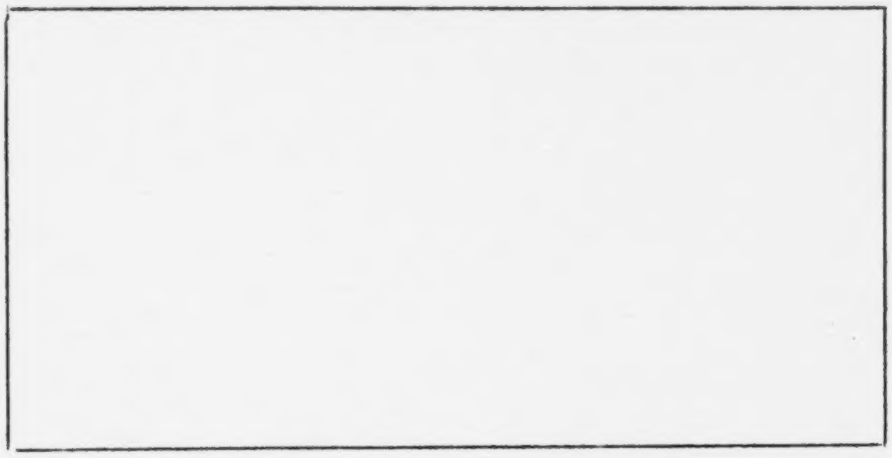
1. All the area we want to measure must be covered.
2. None of the area units we use may overlap each other.
3. The area we want will be the number of area units we used. (That is, simply count the area units needed to cover the area being measured.)

Our first task will be to choose a unit of area. Clearly there are many shapes we could choose from. We could choose any of the following: rectangle, square, triangle, circle, etc..

I. We will begin this search for a unit of area by trying different little "chunks" of area. Naturally, we will choose a shape for our unit area which makes measurement of area easiest..

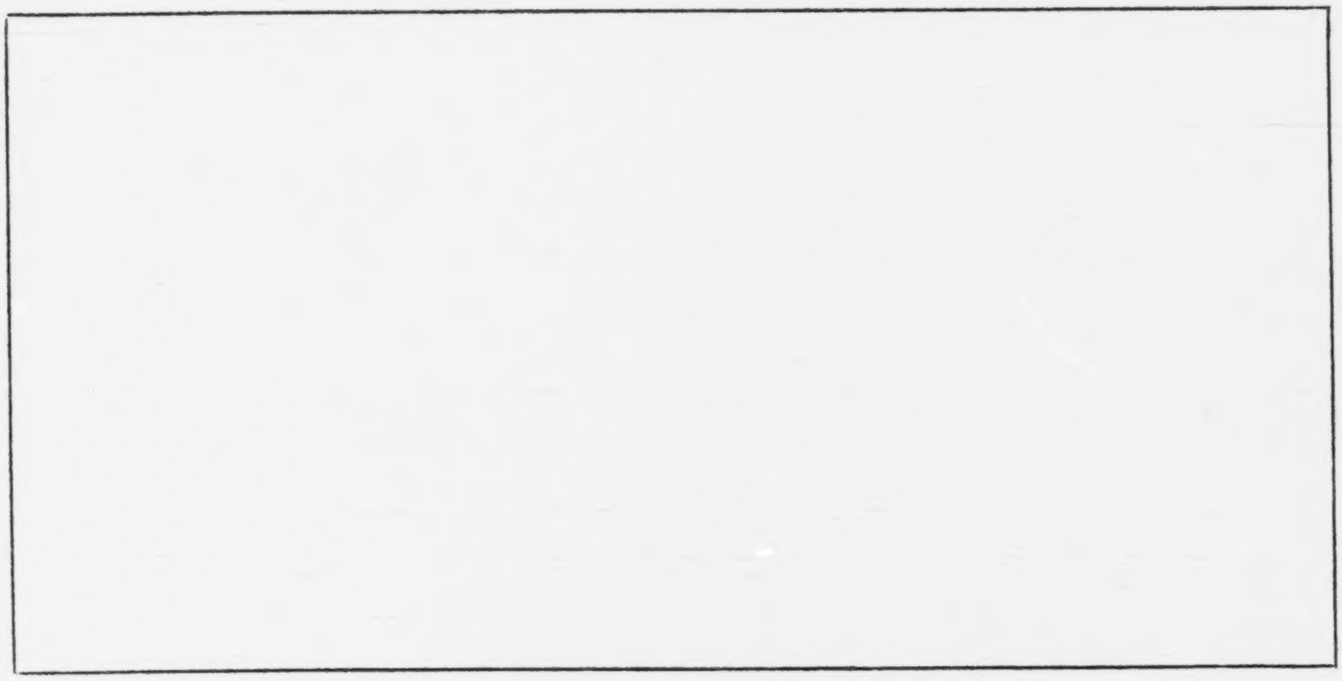
Using the little rectangle (not the square) as a unit of area, find the areas of each of the following larger areas:

(a)



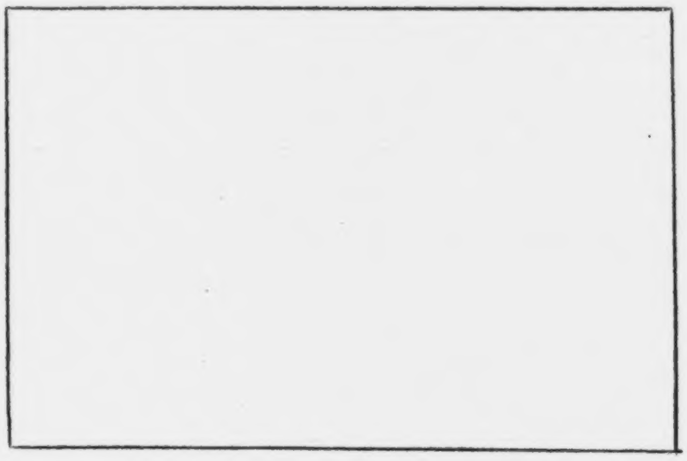
What is the area of the figure above?____________

(b)



The area equals ______________ rectangular units.

(c)



The area equals ________________ rectangular units.

Perhaps you noticed that in our first example we could place the rectangles in two different ways without any trouble fitting them into the area to be measured.

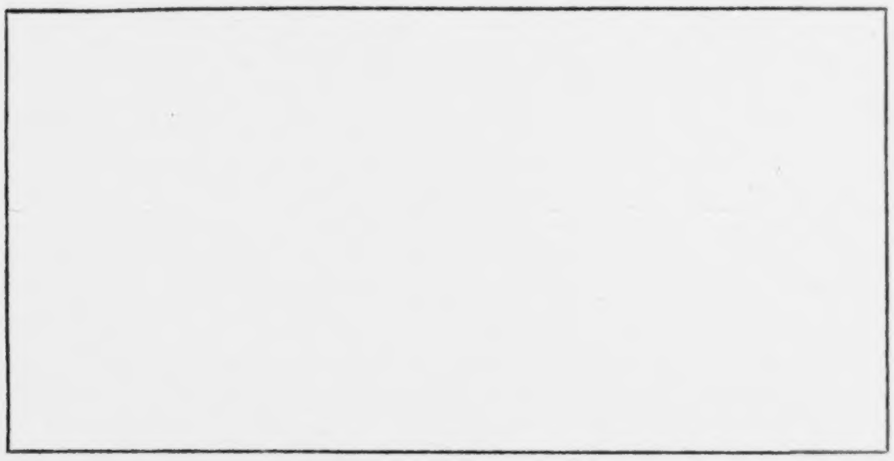
Can we place the rectangles different ways in example (b)?___

Can we place the rectangles different ways in example (c)?___

You very likely agree that in examples (c) and (b) we must be somewhat careful about how we place the rectangles. It doesn't work all ways. In the following examples we shall try different possible units of area to determine if any seem better than the rectangle.

II. Using the small square as the unit, find each of the following areas:

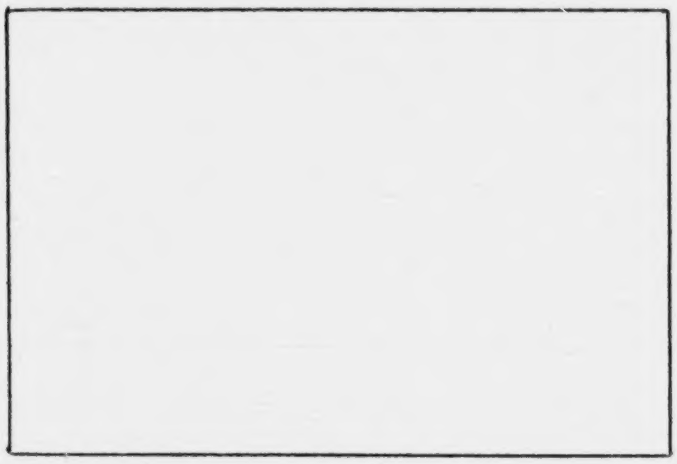
(a)



What is the area of figure (a)? ____________

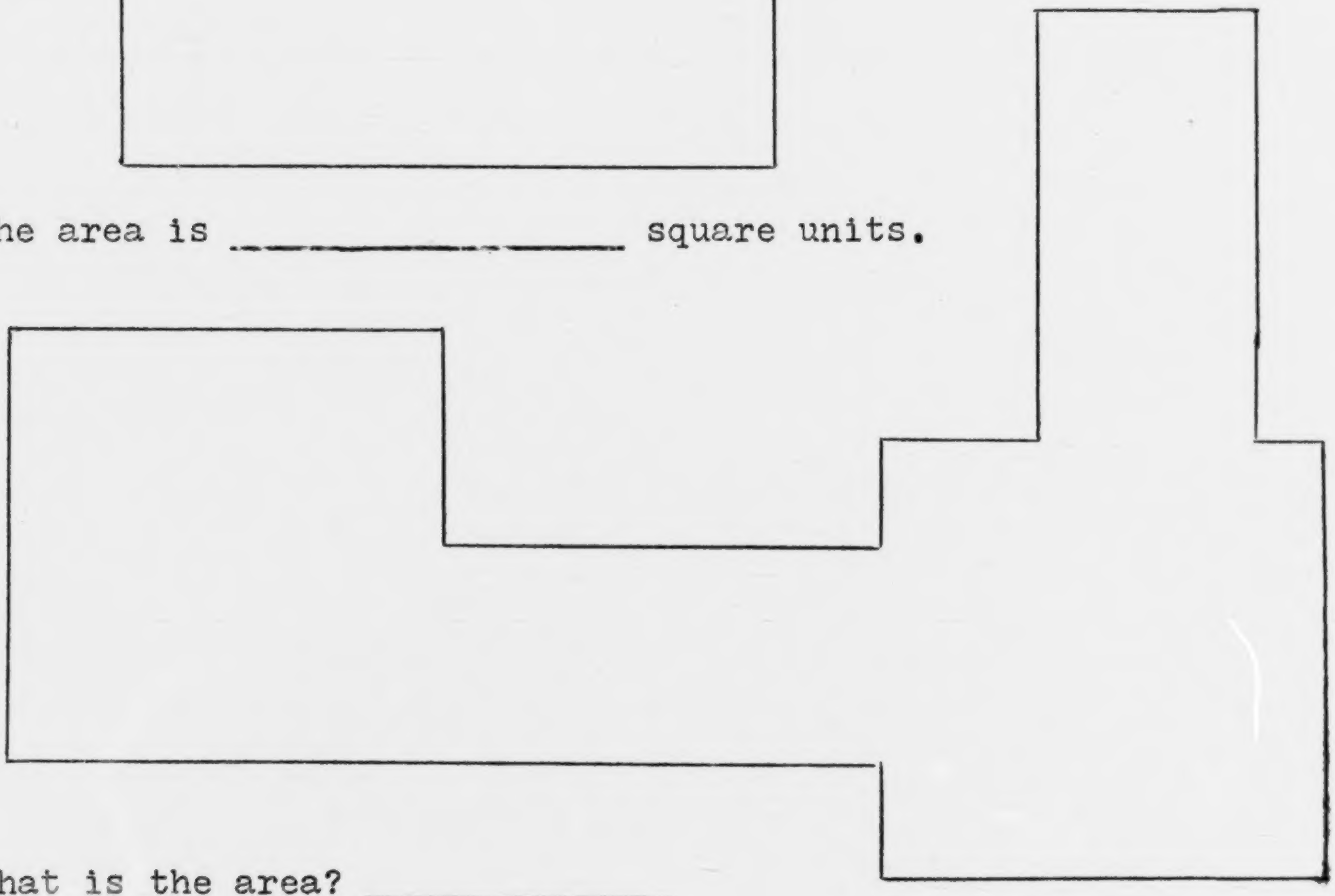
Can we place the square any way we please? ________

(b)



The area is ______________ square units.

(c)



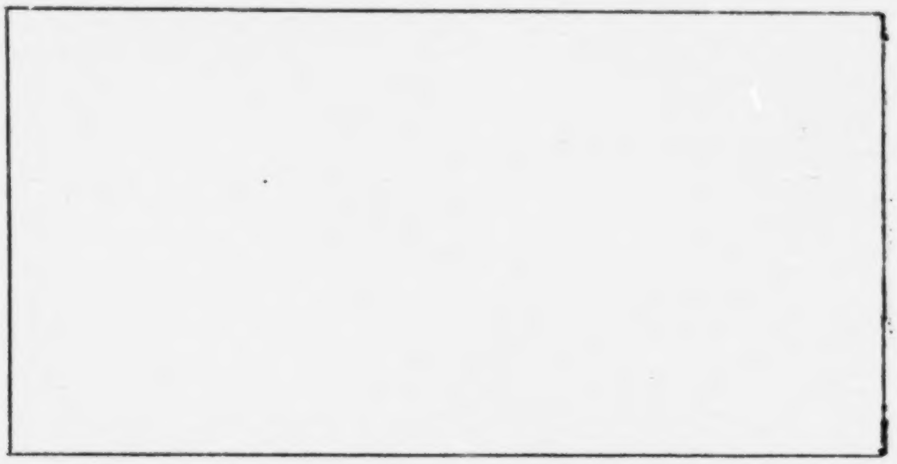
What is the area? ___________

III. On the last page we noticed that there was very little need to worry about how we placed the square. It appears that the square is a convenient unit of area to use, at least more convenient than the rectangle which is not square.

However, before we make any quick decisions, we should investigate several other possibilities.

Find the area of the following figures using the unit area which is a triangle:

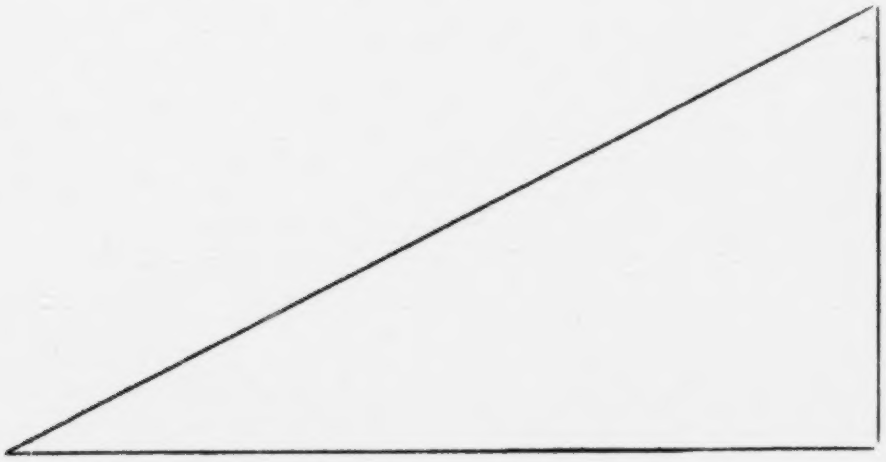
(a)



What is the area of the figure? ______________

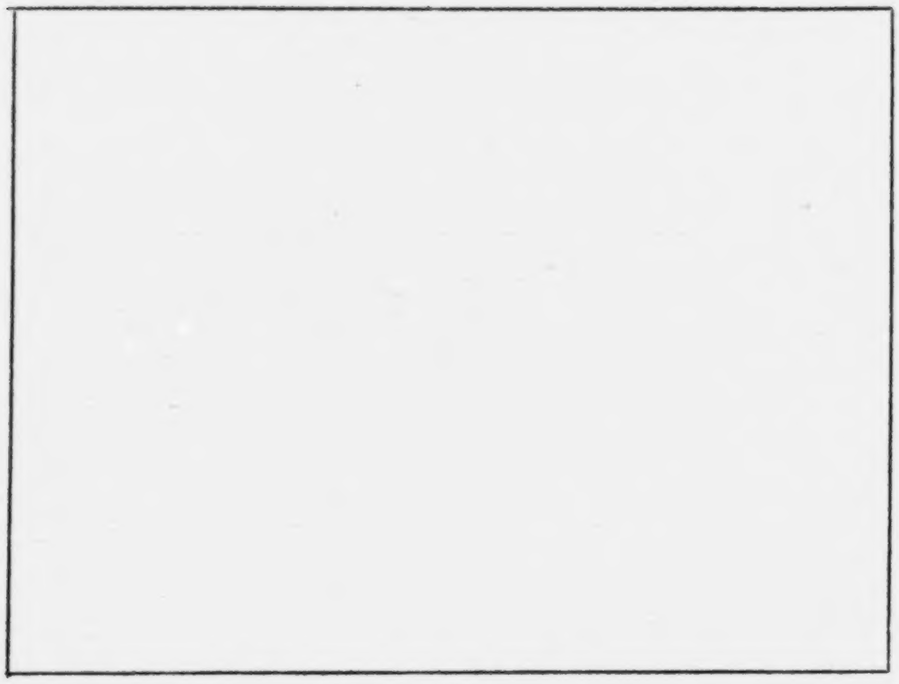
Do we need to be careful in placing the triangles? ________

(b)



The area is ___________ triangles.

(c)

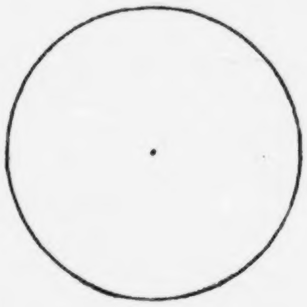


The area is ____________________.

Must we be careful as to how we begin placing the triangles in examples (b) and (c)? ___________

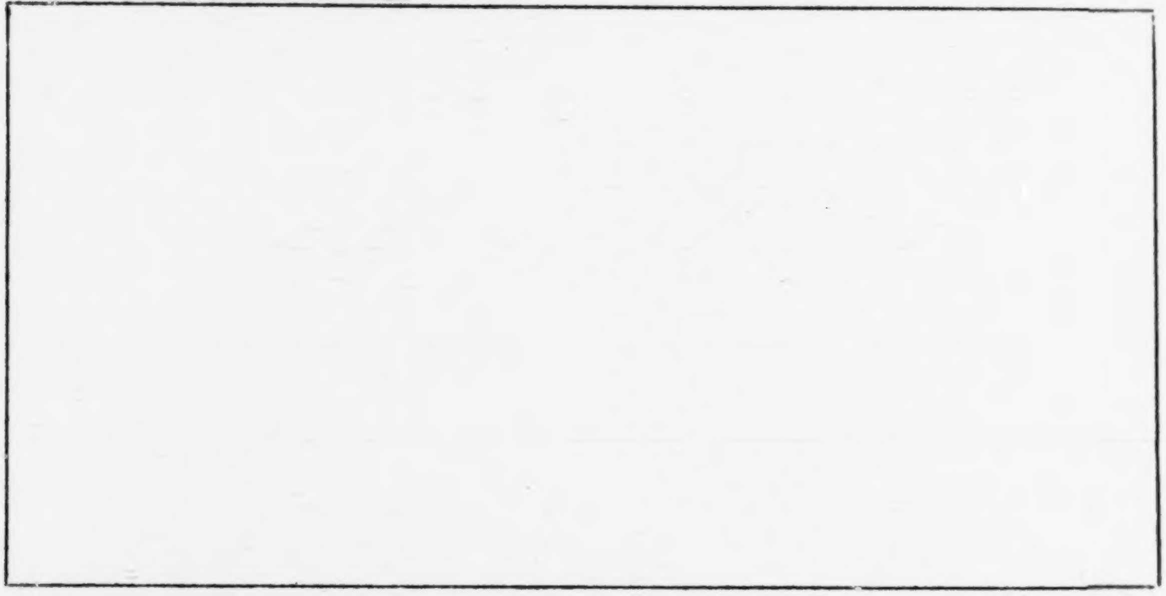
IV. We shall try one more possible shape for our unit of area, the circle. Using the unit area in the shape of a circle, find each of the following areas:

(a)



The area is ____________ unit(s).

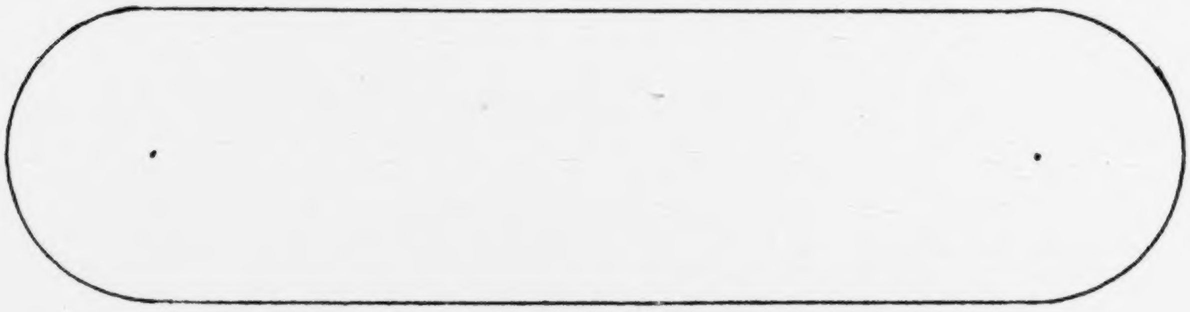
(b)



The area is ___________ unit(s).

Is the circle inconvenient for any reason? _______________

(c)



The area is _________ unit(s).

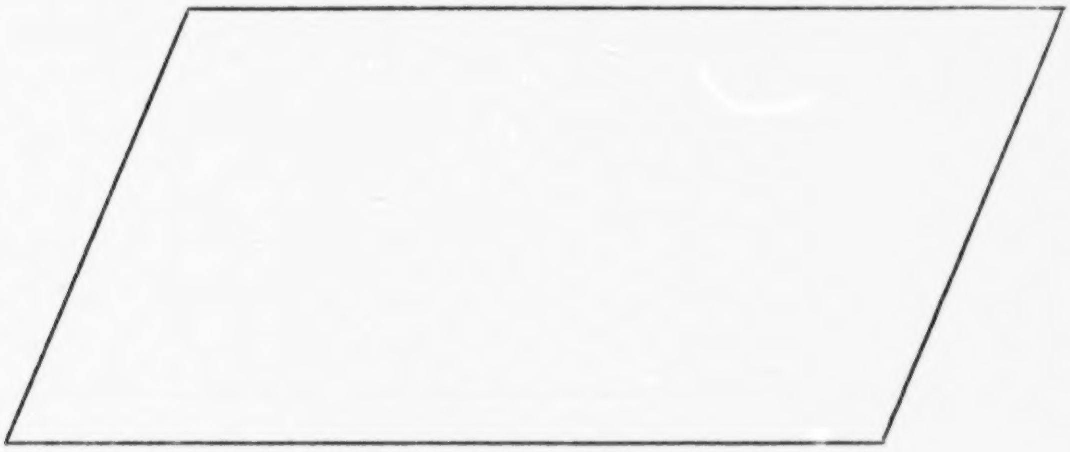
Are there still difficulties? ___________________

V. At this point you have probably decided that the square is the most convenient shape to use as a unit of area. It has two very desirable features:

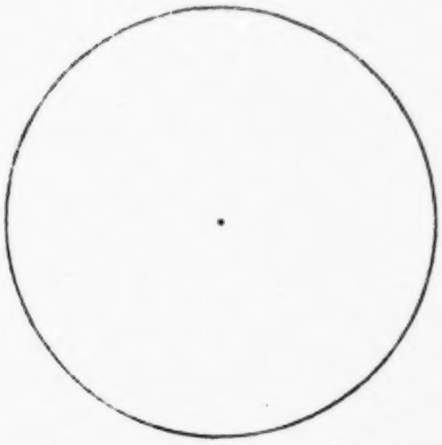
1. Squares fit very nicely next to each other.
2. It makes no difference how we place the squares since they are as long as they are wide.

The square is not without its problems. Look at the following figures and try to find their areas using the square:

(a)



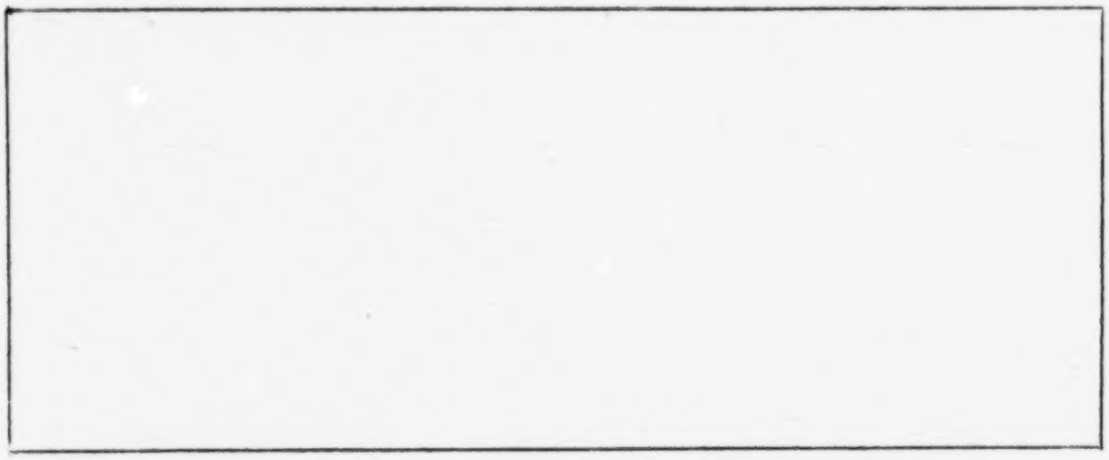
(b)



Although the square doesn't make all area problems easy, we'll decide to keep it as our unit just the same. In later sections of this booklet we'll try to think of clever ways to find areas of shapes which can't be covered neatly by squares.

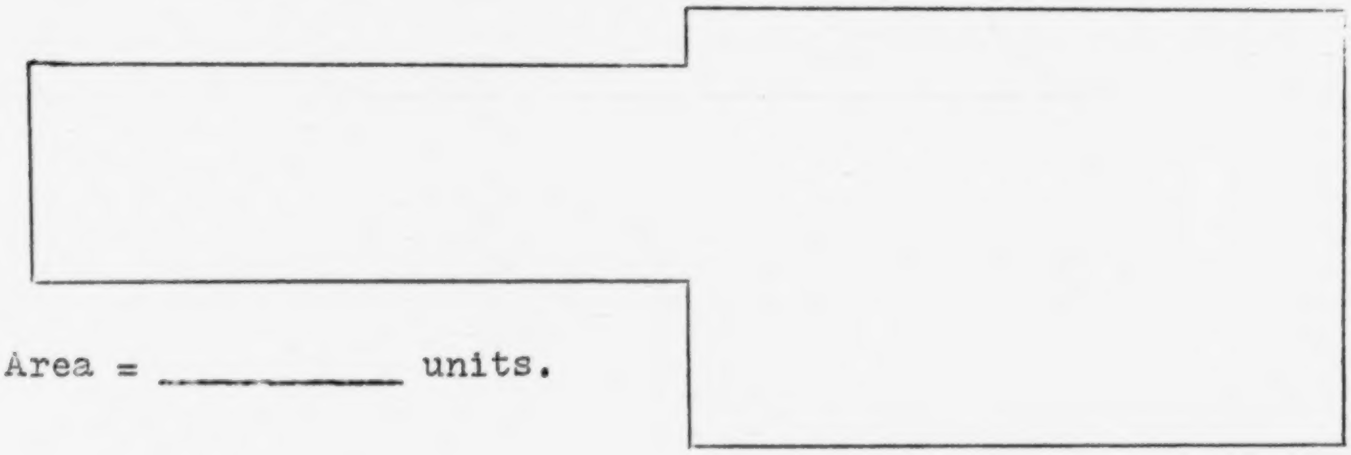
VI. Using the square unit of area, find the areas of each of the following figures:

(a)



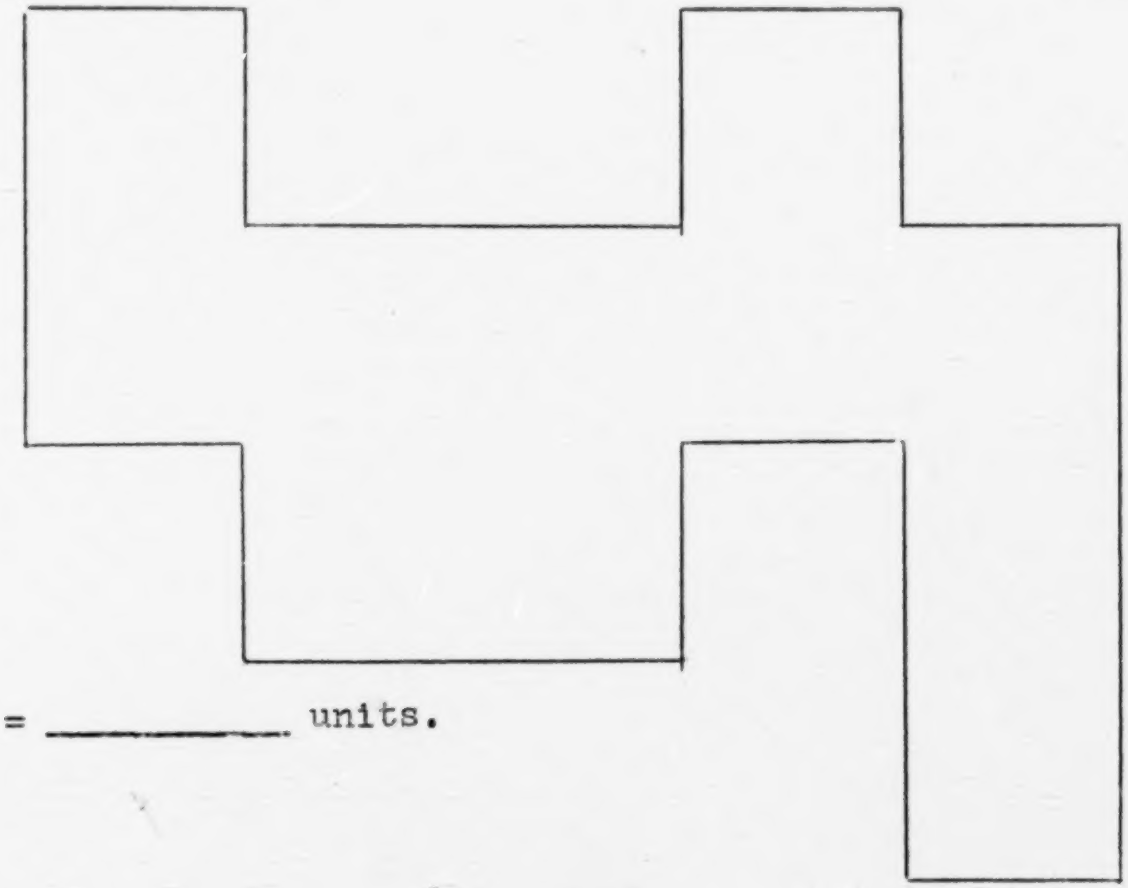
Area = ____________ units.

(b)



Area = ____________ units.

(c)



Area = ____________ units.

PART 3

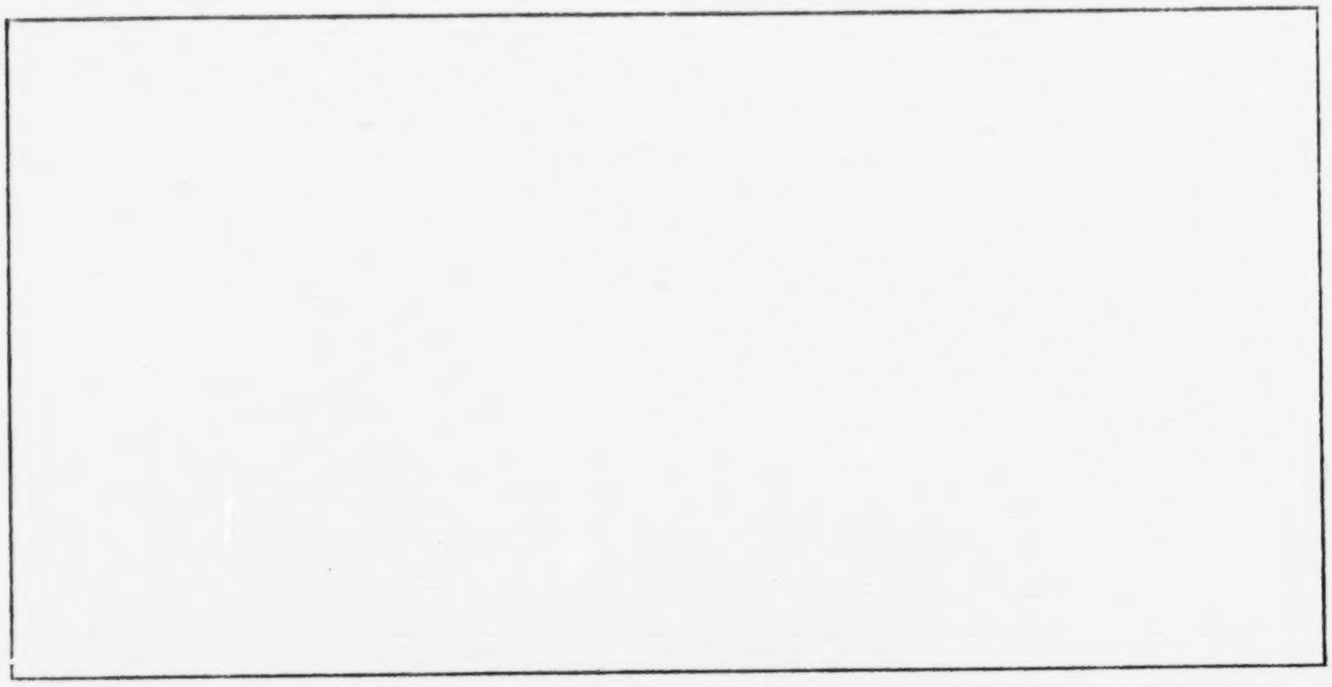
Area of a rectangle

So far we have chosen a shape for our unit of area and have gained a pretty good idea about what has to be done to find the area of a figure.

We are now ready to begin looking for faster methods for finding areas of figures of different shapes. We will begin our study by looking at rectangles.

With the square unit of area, find the areas of each of the following figures and answer the questions which follow. (You should have 20 squares to work with.)

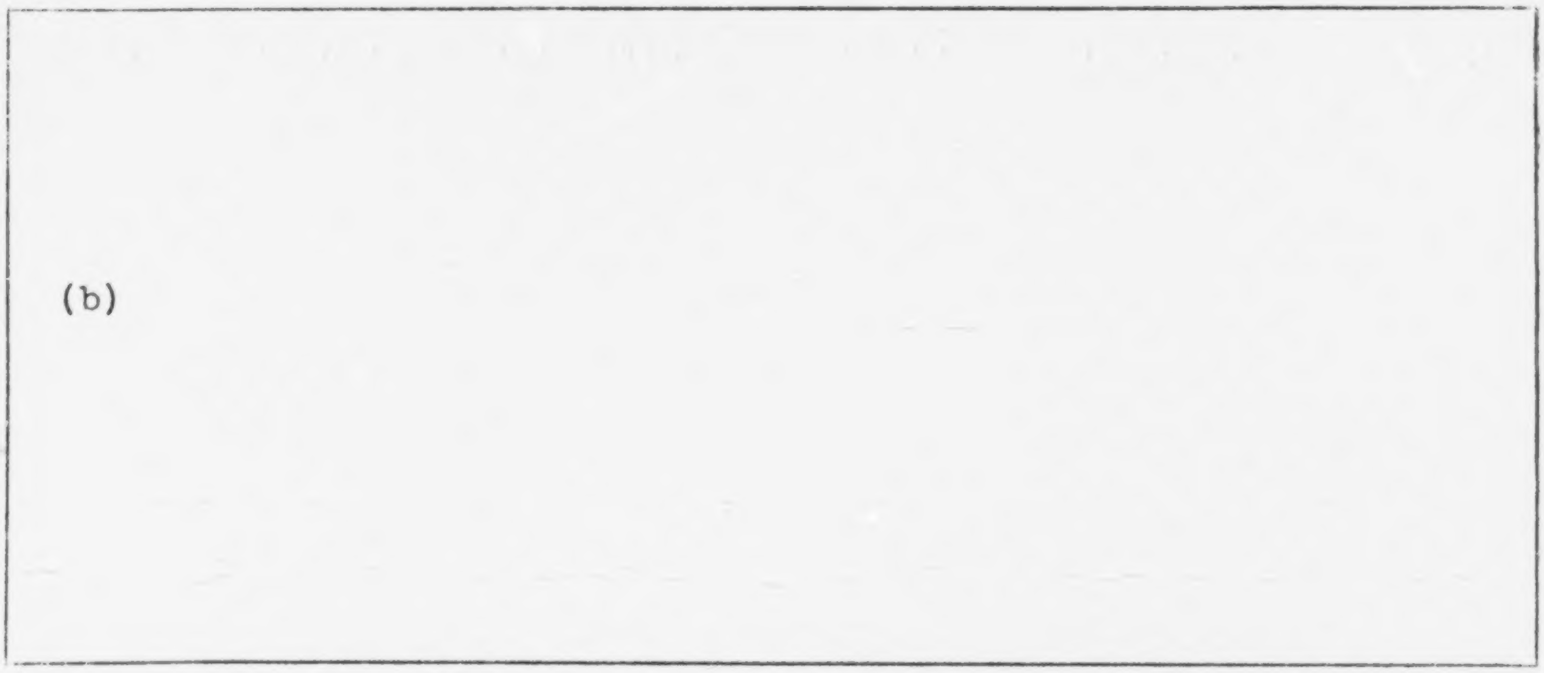
(a)



The area = __________ squares.

How many rows of squares are there? ____________

How many columns of squares are there? ____________


(b)

Area = __________ units.

How many rows of squares? ____________

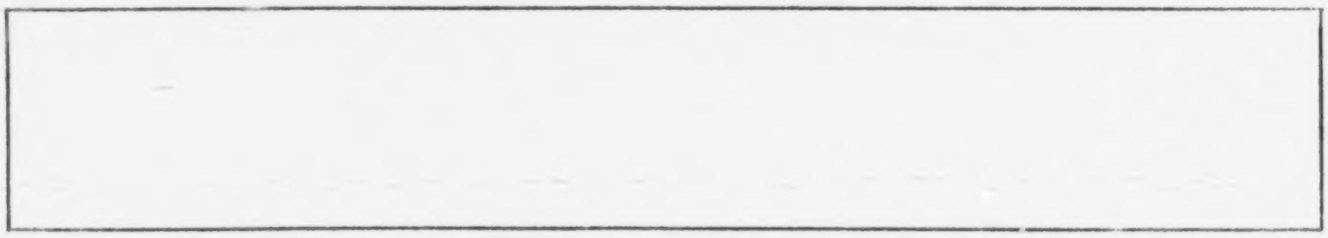
How many columns of squares? ____________

Were there enough squares with 20 to cover the entire area? __________ What do you do to find the area when there are not enough squares to cover the area? ____________________

You have very likely concluded that we can move the squares and remember what parts of the area we have covered. In fact, you probably realize that if we could remember exactly what area we had covered, we could get by with using only one square. It certainly is more convenient not to carry huge boxes of small squares whenever we have to work an area problem. Perhaps it would be most convenient to use only one square. We could just keep moving it until we had covered the entire figure. The area would simply be the number of times we placed the unit area on a different part of the figure to be measured.

Using just one square, find the areas of the following figures. (If a pencil would help you keep a record of which parts of the figure have been covered, be sure to use one.)

(a)

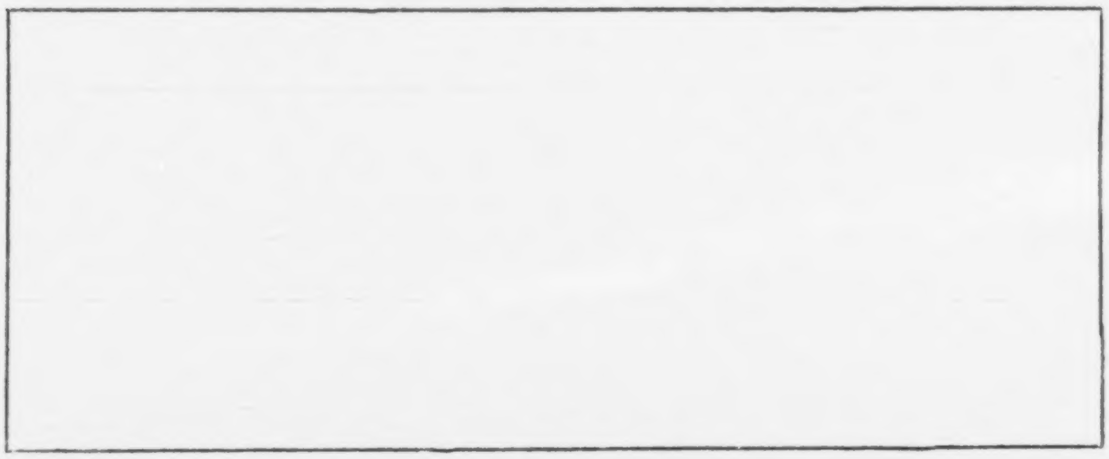


Area = ___________ squares.

How many rows of squares? _____________

How many columns of squares? _____________

(b)



Area = _____________ squares.

How many rows of squares? ______________

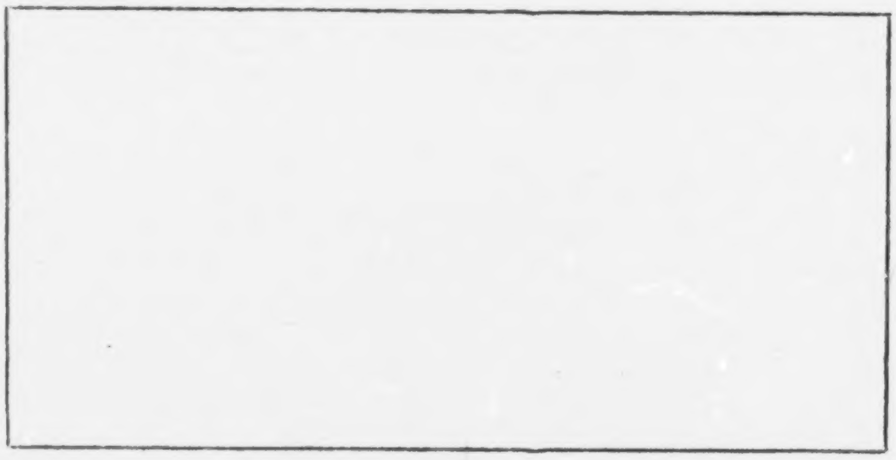
How many columns of squares? _________________

In each of the last examples we have done there have been three questions to answer. The first was to state the area. The second was to state the number of rows, and the third asked how many squares were in each row (how many columns). Looking back we see that if we multiply the number of rows times the number of columns we get the same number as that we found for the area. So it appears that we can cut down on the ammount of work we need to do. We don't have to cover the whole figure at all. We simply need to find a convenient method for finding how many rows and how many columns there would be. We have the following formula:

AREA = (number of rows) times (number of columns)

Earlier we talked about the inconvenience of carrying a box of squares along whenever we need to find area. Maybe it isn't even very handy to carry one square in a pocket. For one thing it is very likely to break whenever anyone sits down too hard, and it isn't the most convenient thing to carry. We shall now attempt to find area by using only a device to measure length. We will have a short "ruler" which is exactly as long as an edge of our unit square.

Try to find the area of the following figure using only the short "ruler". (Remember the formula we mentioned on this page.)



Area = __________ squares.

CLASS DISCUSSION What did you do to find area?

We now see that we can find the area of a rectangle by measuring length. With the very short ruler we used it was necessary to keep flipping the ruler end over end, first along the top to find the number of columns and then along a side to find the number of rows. Multiplying these two numbers together gave us the area.

Now take out a ruler (a foot marked off by inches) and find the area of the same figure. You will find that the ruler is long enough so that we don't have to keep flipping our measuring device end over end. If we had to measure very big rectangles, we would very likely use a tape measure like ones carpenters use.

We also notice that the number of columns is simply the "length" of the rectangle, and the number of rows is the "width" of the rectangle. We can now restate our formula as follows:

AREA = Length times Width

If we wish to become still more brief, we can agree to let the letter "l" stand for length and the letter "w" stand for width. Letting the letter "A" mean area, we now have:

$A = l \times w$ ("×" means "times")

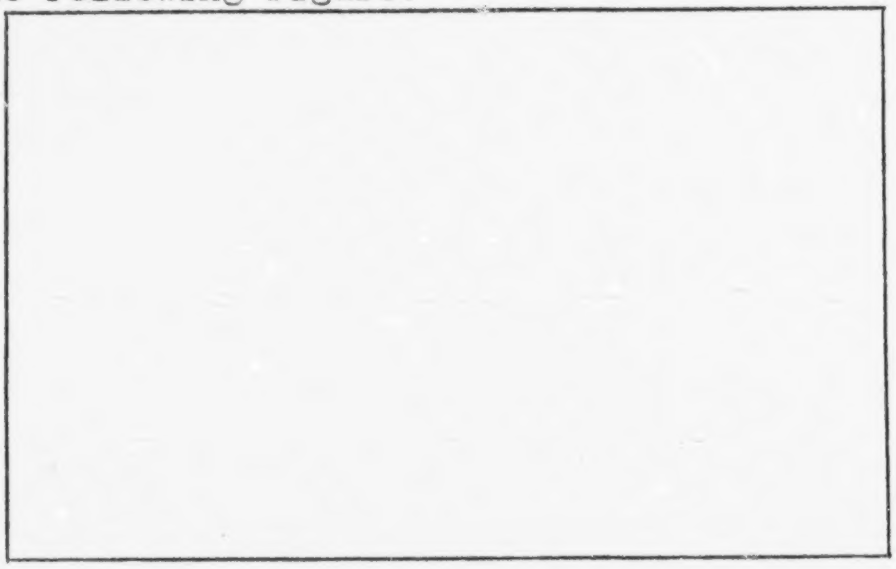
This formula allows us to measure length and width to find out how many squares it would take to cover a rectangle. It makes area much easier since rulers are easier to handle than squares.

Up to this point we have all been using the same size square. We have expressed area as simply being a number of squares. We have used a square which is one inch long and one inch wide. But our small square isn't always convenient. If you look in an encyclopedia you will find that the area of a state is usually given as a number of "square miles". This means that the unit of area being thought of is a huge square a mile long and a mile wide. On floor plans for houses and other buildings the floor area is usually given as a number of "square feet". This means that the unit of area being used is a square a foot long and a foot wide. We have been working with the "square inch".

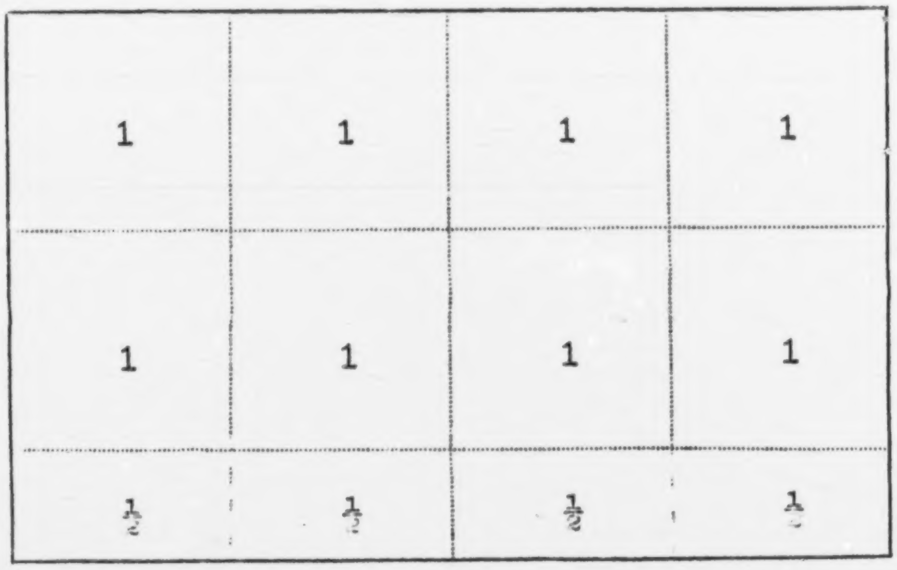
We measure length using many different units such as foot, centimeter, yard, furlong, or mile. Units of area have the same variety such as square foot, square centimeter, square yard, square furlong, or square mile.

When telling someone about how much area we have it is very important always to also tell what unit of area measurement we are using. If a floor is 10 feet long and 13 feet wide, the area is 130 square feet, not simply 130. If a ranch is 5 miles long and 6 miles wide, the area is 30 square miles, not simply 30. Just to give a silly example of what could happen if no one cared about what unit was being used, a contractor might tell a customer that he will build a house for $10,000 with 1,500 units of floor space. After the contract is signed the contractor might build a beautiful doll house with 1,500 square inches of floor space.

Looking back at the problems you worked so far you will notice that the length and width always came out very nicely. Using the small unit square once again (the square inch) try to find the area of the following figure:



You will notice that it is impossible to cover the figure completely without either cutting the figure and changing its shape or cutting unit areas and placing the pieces in a way which will cover the figure. No matter which of the methods you choose, you will find that the area is 10 square inches.



The happy thing about this example is that the area is still length times width. ($2\frac{1}{2}$ times $4 = 10$)

SUMMARY (area of a rectangle)

1. Finding area is really a matter of finding how many unit areas will fit on the figure we wish to measure.

2. In the case of rectangles (squares are also rectangles) we can measure length and width and multiply the two numbers to obtain the area.

3. We must always be careful to tell what unit of area we are using.

4. Even though the length and width are not whole numbers, our formula $(A = l \times w)$ still works.

EXERCISES (be very careful to watch which units you use)

1. Find the area of a rectangle whose length is 10 inches and width is 8 inches.

2. Find the area of a square ranch which has a side 3 miles long.

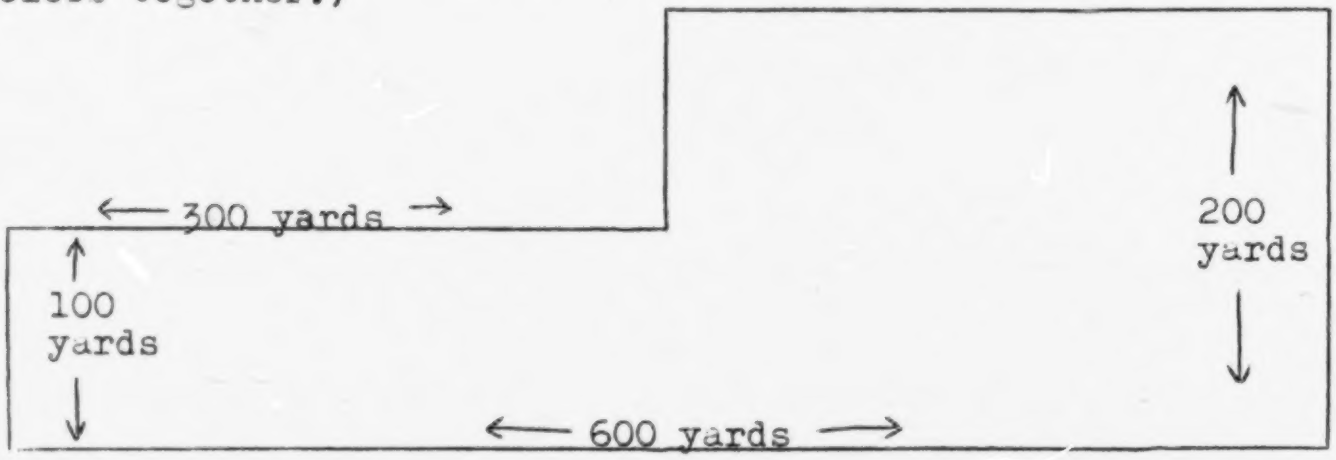
3. Find the area of a rectangle whose length is 4 inches and width is 5/8 of an inch.

4. A rectangular field has an area of 450 square yards and a length of 30 yards. How wide is the field?

5. Jack wanted to know the area of the grass he would have to cut. The yard was 120 feet long and 60 feet along the street. In the center of the lawn his mother had 60 square feet of flower garden. The house covered an area of 980 square feet. What was the area of the lawn?

6. Find the area of a slab of concrete which is 3 yards long and 2 feet wide.

7. Find the area of the field pictured by the following diagram. (Note: You may wish to think of it as several rectangles placed close together.)



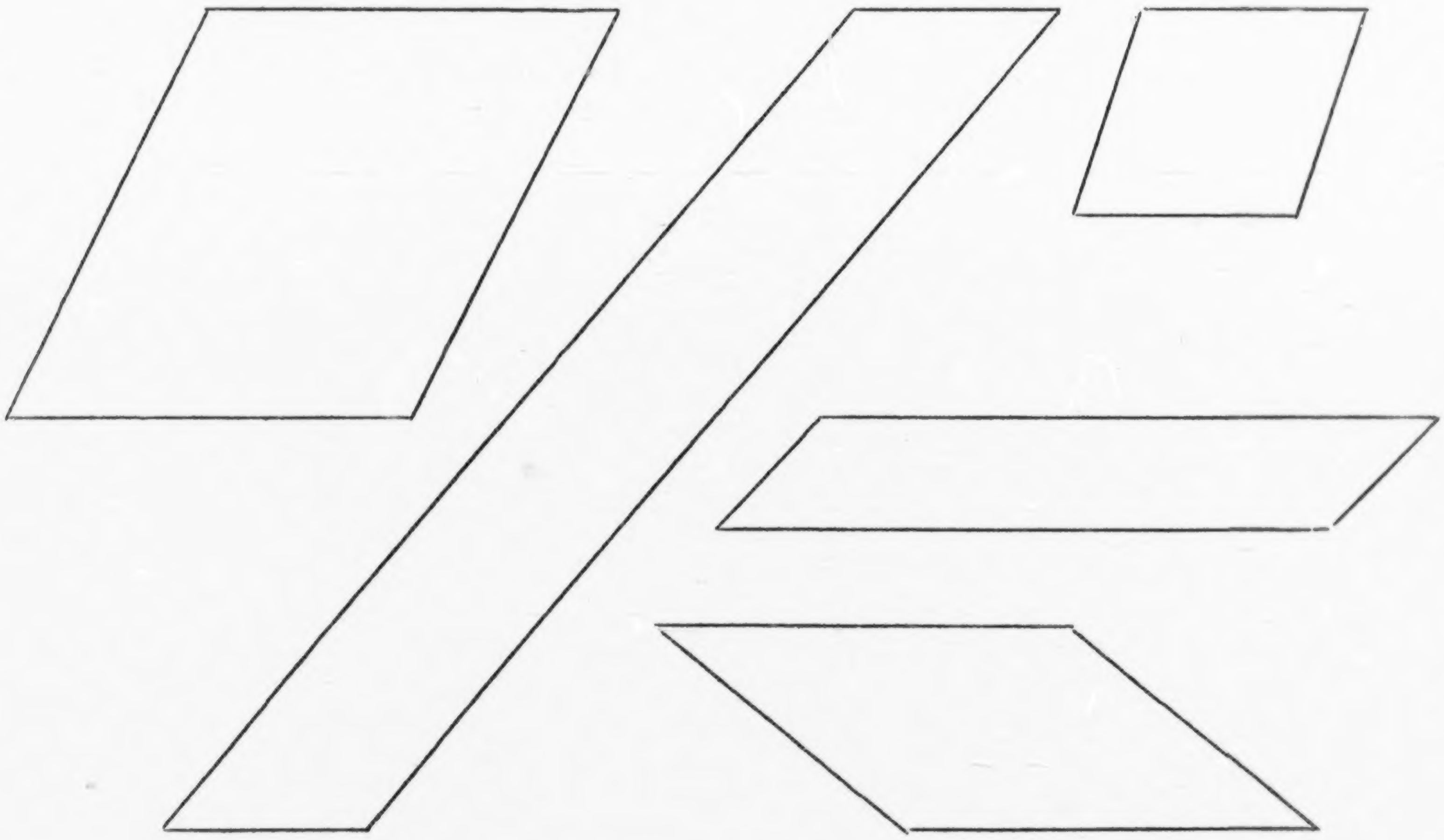
PART C

Area of a parallelogram

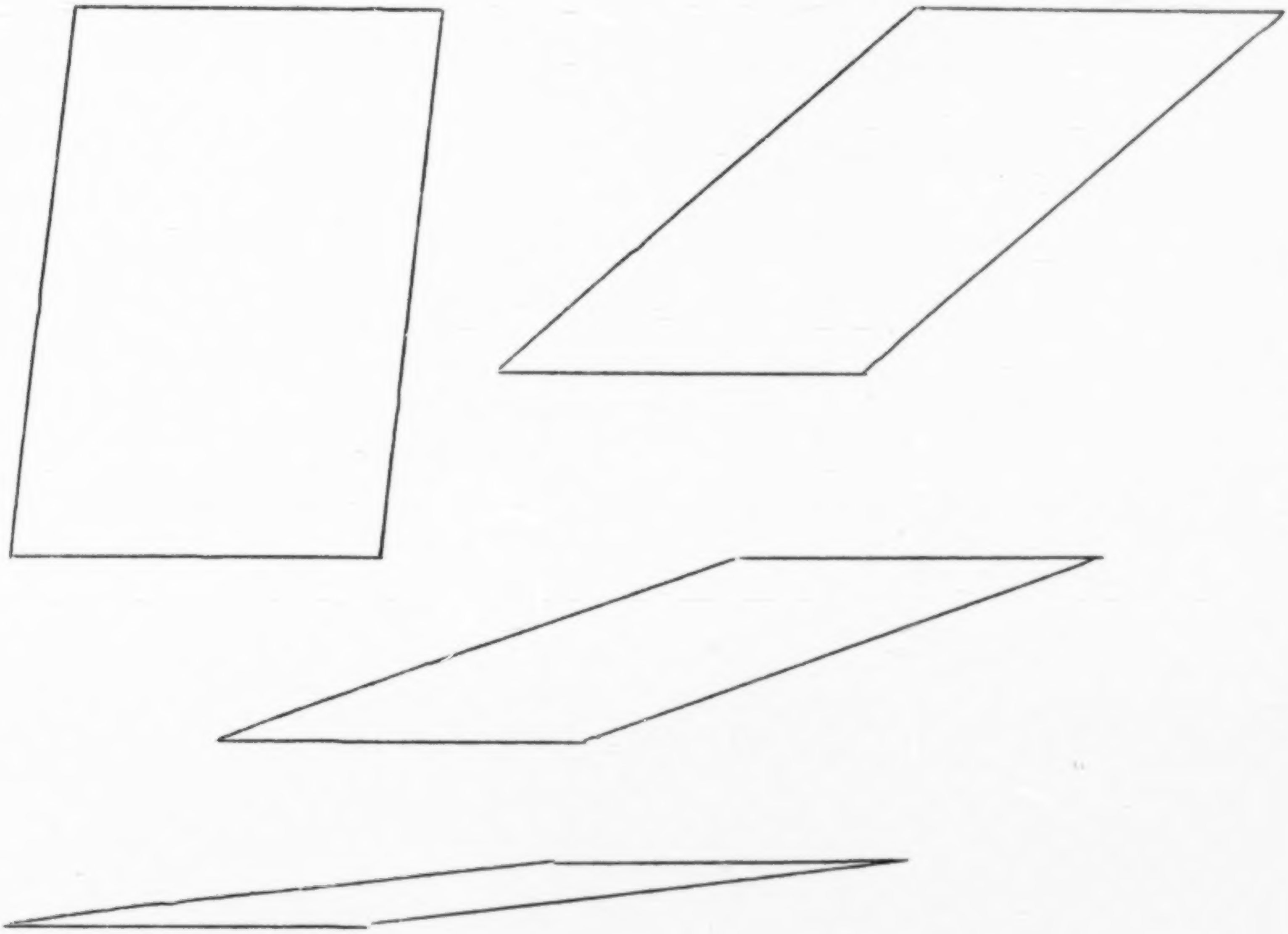
Up to this point we can find the areas of rectangles and squares. We have also found an equation or formula that helps us find these areas by a very simple procedure.

But there are other rather common figures whose areas are often wanted. Closely related to a rectangle, but not a rectangle, is the parallelogram. If we want to describe a parallelogram carefully, we would have to say something like this: "A parallelogram is a four-sided figure with two pairs of parallel sides." A poor definition, but one which may be helpful, might define a parallelogram as "a rectangle which has had an accident

The following are parallelograms:

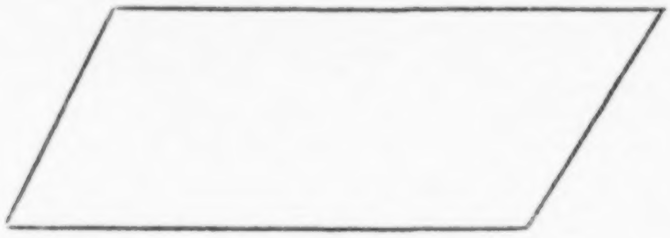


It would be very convenient if we could simply measure two of the sides of a parallelogram and multiply to find area as we did with rectangles, but sadly this does not work with parallelograms which are not rectangles. The following pictures should convince you that the area of a parallelogram depends largely on just how much it has been "pushed down" from being a rectangle. All the following parallelograms use the same lengths for sides.



In the next pages we will try to think of a sneaky way to find the area of these figures. If we can, we'll use what we already know about rectangles.

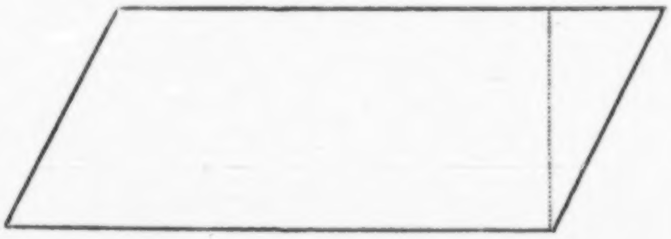
First, we will list some of the things all parallelograms seem to have in common:



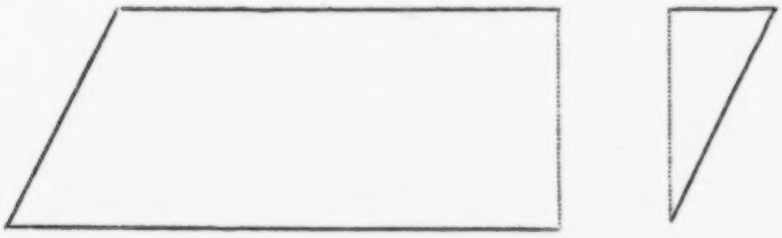
A parallelogram is a four-sided figure which has:

1. opposite sides equal and parallel
2. opposite angles equal
3. the sum of all angles equal to 360 degrees

If we cut the parallelogram and move parts of it around we will not change the area, only the shape. We begin by drawing a dotted line from one of the wider angles directly to the longest side that doesn't help make that angle.

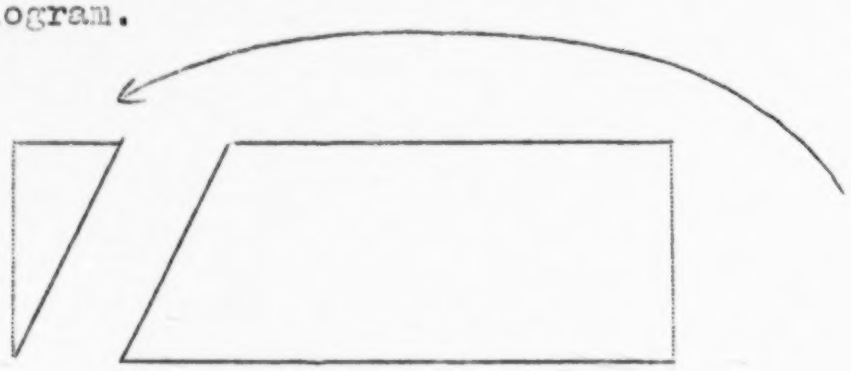


Next, we cut the parallelogram along that dotted line.

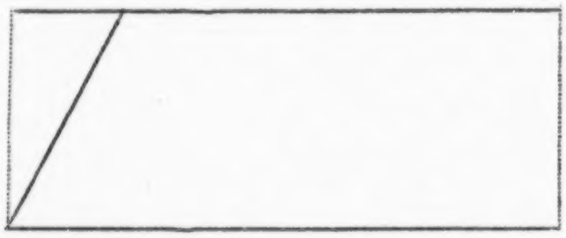


(You will notice that if the dotted line is made as short as possible, cutting the triangle away from the parallelogram leaves behind something which looks very much like the end of a rectangle.)

For our next step we will move the triangle to the other end of the parallelogram.



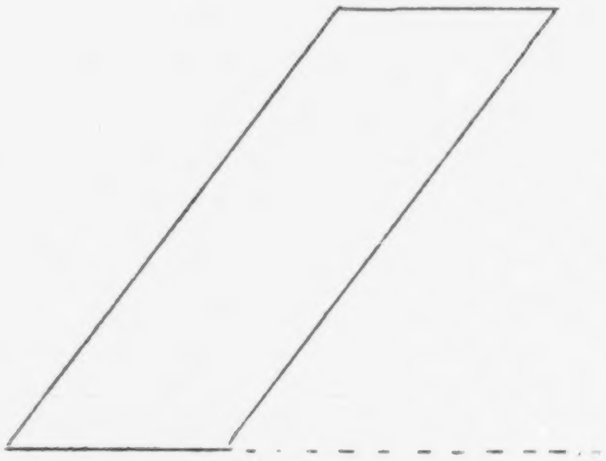
Putting it in place, we see that we simply have a rectangle whose length is the bottom of the original parallelogram and whose width is the dotted line.



So far it appears that we have a formula for finding the area of a parallelogram. If we decide to call the bottom side the "base" and call the distance from the base to its opposite side the "height," then AREA = Base times Height.

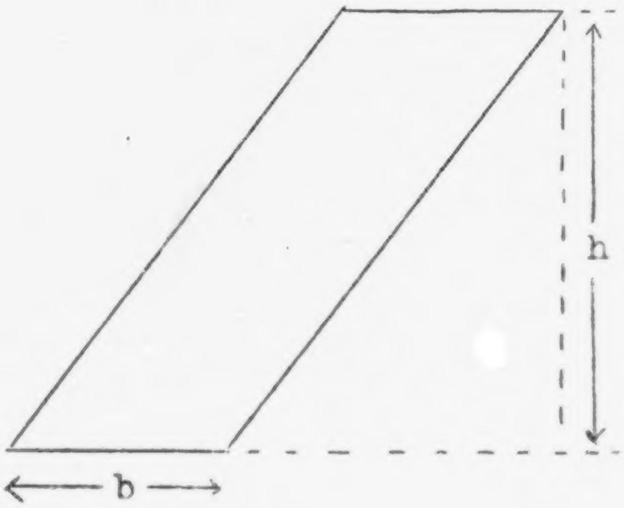
(Note: There is really no good reason to talk about base and height instead of calling the two measurements length and width. Our only reason for doing this is that many mathematics books do it this way. If we wanted to talk about the base and height of a rectangle, our formula would become AREA = Base times Height.)

Before declaring that we have the correct formula, we should look at a few somewhat strange parallelograms. Some parallelograms have a very short base and are very tall:

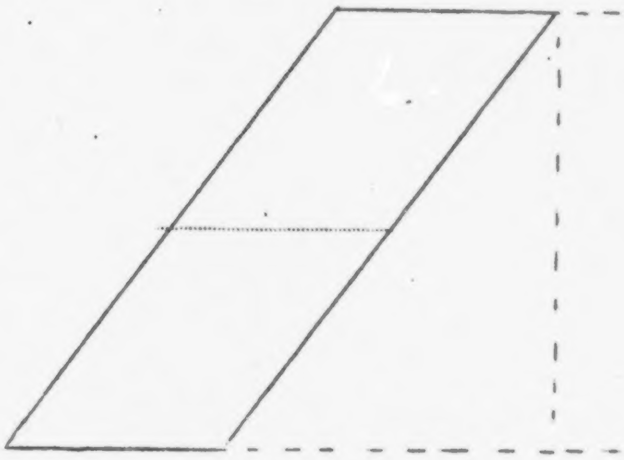


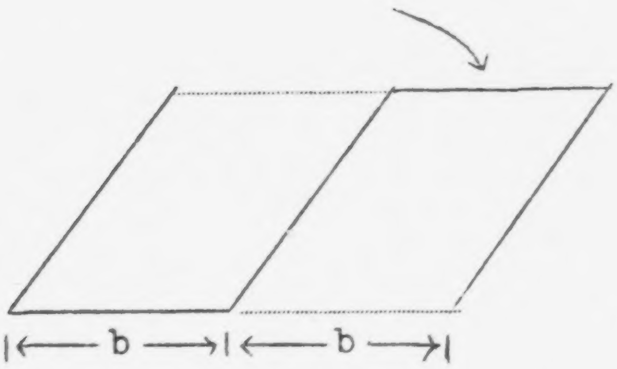
In such cases it is not always possible to draw a line perpendicular to the base which goes to the opposite side. To get a good idea as to what the height should be, we imagine an extension of the base which goes far enough to come beneath the highest sharp angle of the figure.

The height is then the distance from this angle to our extended base line.



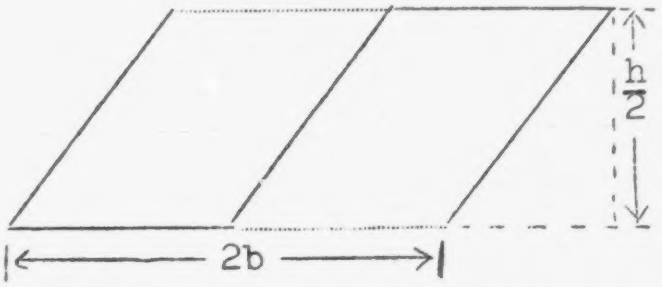
Next, we cut vertically through the parallelogram to make two figures, both parallelograms, which look exactly alike.





We then place the two figures along side each other. We now have a new parallelogram. Its height is half the height of the one we began with and its base is twice as long as the base of the one we began with. The important thing is that our cutting and re-shaping doesn't lose or gain area.

Our parallelogram now behaves nicely as the first parallelogram we looked at. To find area we simply have to multiply base times height.



The base is 2 times "b", and the height is half of "h". Multiplying, we get AREA = $2b \times \frac{h}{2}$ which is just $b \times h$, exactly what we hoped for.

There are worse examples that we might have chosen. The Parallelogram could have had such a short base and could have been so tall that even after making two parallelograms and putting them together the new parallelogram would still be too high to handle like our first example. In that case we would then cut again and do the whole thing over; after the second cut we would have a parallelogram with base 4b and height $h/4$. If that one were still too tall and thin, we would do the same thing again.

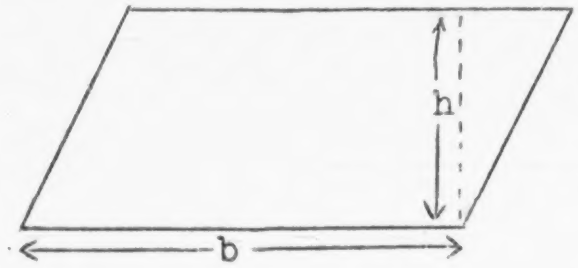
CLASS DISCUSSION: Is it possible to have a parallelogram so tall that it would be impossible to cut often enough to make the result look like the example on page 22?

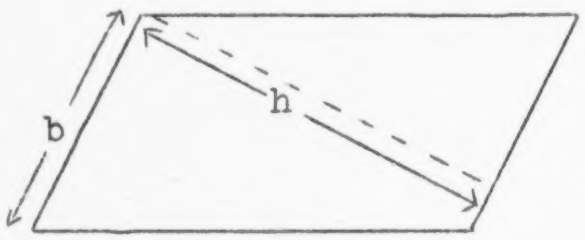
We haven't proved that our formula for a parallelogram is correct, and we certainly haven't looked at all possible shapes and sizes of parallelograms, but the evidence in favor of our formula looks very strong. So, we shall declare that the rule for finding the area of a parallelogram is the following:

AREA = Base times Height

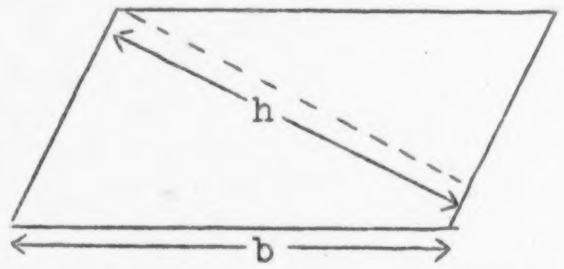
$A = b \times h$, if we prefer a statement that looks brief.

Before going on to problems, we should look at one more fact about our formula. It turns out that it doesn't make any difference which side one chooses to call the "base". (If it did make a difference it would be shocking indeed.)





The following two pictures show different choices for "base" and "height". Both choices give the same answer. (If you have doubts, you should measure in both pictures with your ruler and multiply to see that the answers are the same. If the answers are a wee bit different, you may blame your ruler, not the formula!)



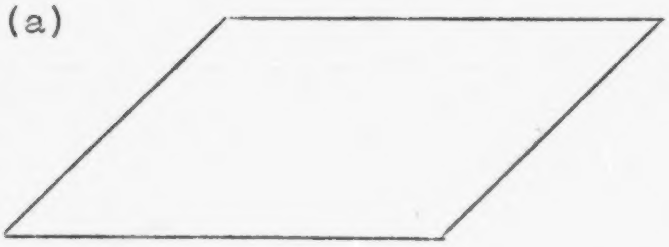
Equally important is to remember that once you have selected a "base" you have just one choice for "height". The person who worked the problem above will come out with a rather sour result.

EXERCISES

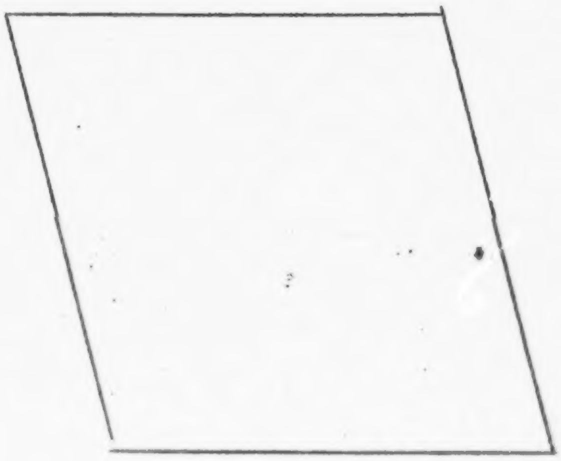
1. Using your rulers, find the areas of the following figures.

(If you know how to construct perpendicular lines for the "height" and if you have the necessary tools, you will perhaps get closer results. If not, you should consider your answer correct if it comes fairly close to what your teacher tells you.)

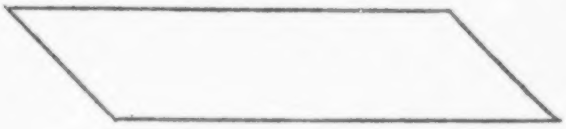
(a)



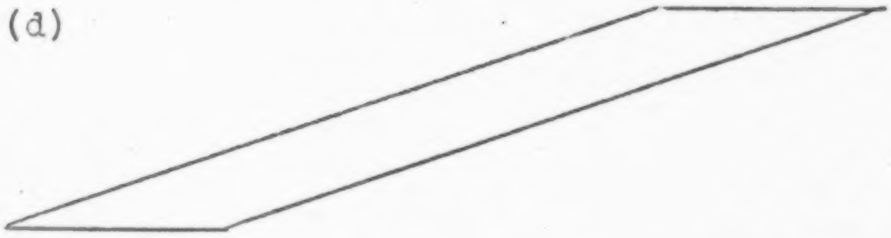
(b)



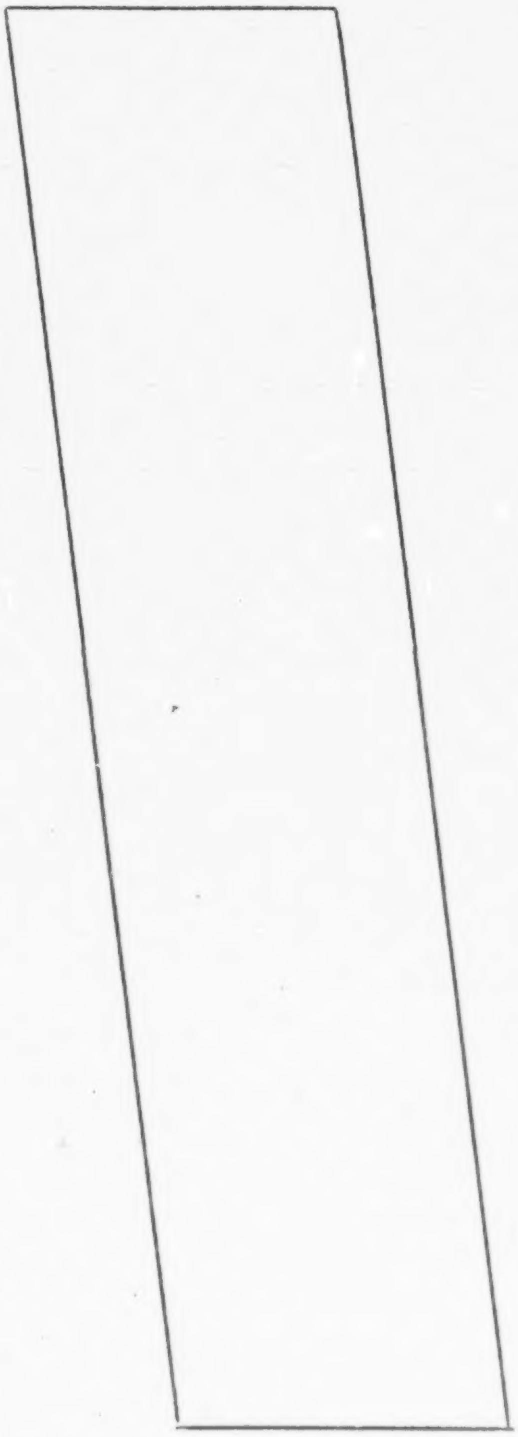
(c)



(d)



(e)



2. Find the area of a parallelogram with base 35 centimeters and height 7 centimeters.

3. Find the area of a parallelogram with base 3 fathoms and height 6 fathoms.

4. Find the area of a parallelogram with base 2 feet and height 13 inches.

5. A parallelogram has an area of 36 square yards and its base is 4 yards. How high is the parallelogram?

6. The city of Sandblast was staked out during a severe dust storm by a city engineer whose equipment had blown away. After the city was built it was noticed that all the streets were parallel with those a block over but that every block was in the shape of a parallelogram. Each block had an area of 5,000 square yards, and each block was 100 yards long. How "deep" was a block in the city of Sandblast?

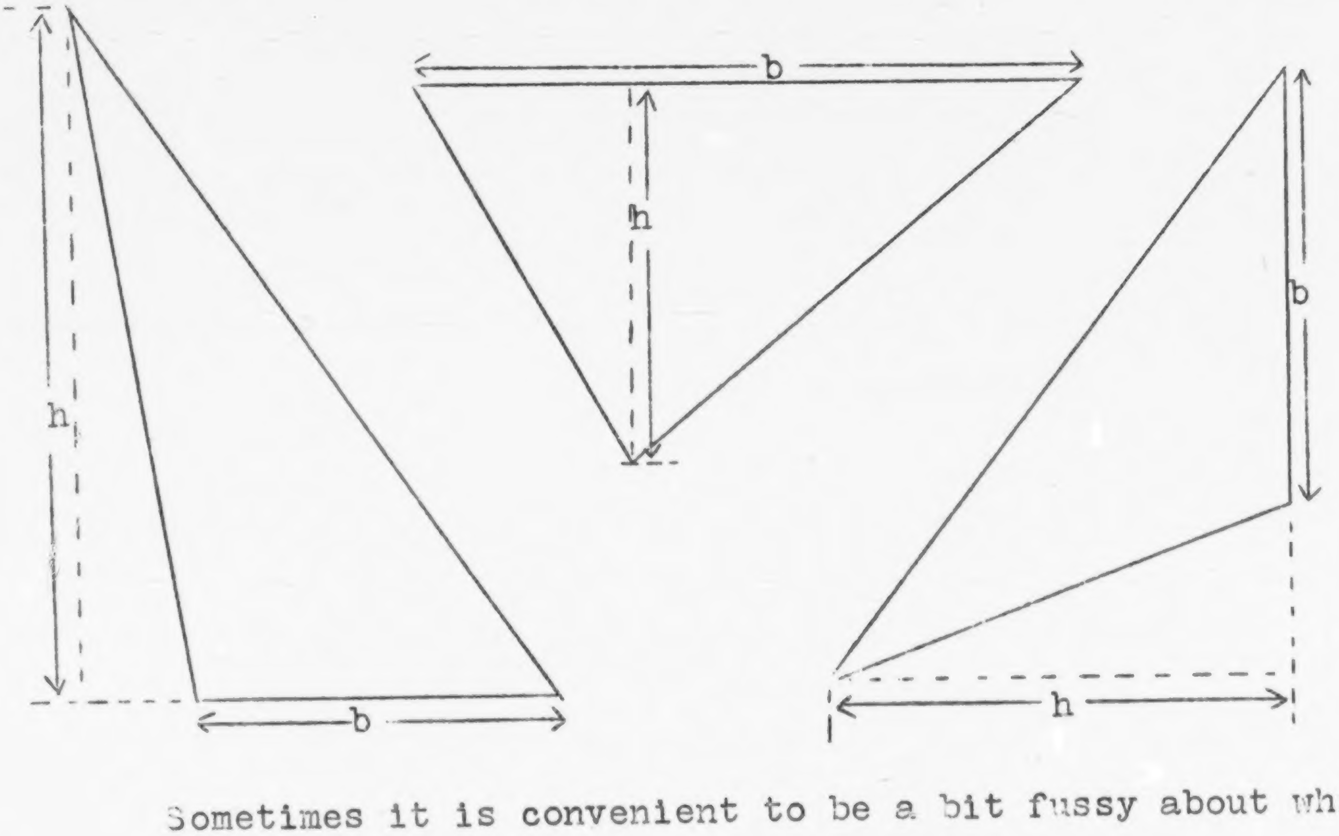
7. CLASS DISCUSSION If four sides, two of them 10 feet long and two of them 5 feet long, are used to make a parallelogram, what is the most area the parallelogram could have? ____________
what is the smallest area the parallelogram might have? _________

8. CLASS DISCUSSION If you know how to find the area of a parallelogram, is there any need to remember the formula for finding the area of a rectangle?

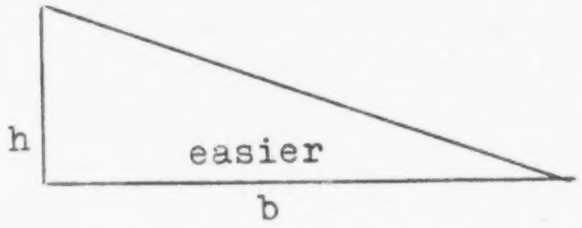
PART D

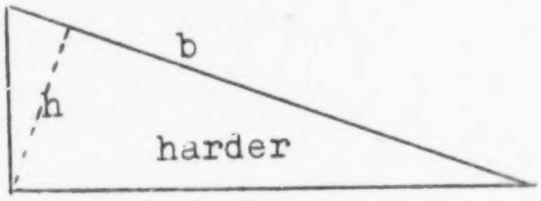
Area of a triangle

Finding the area of a triangle will become surprisingly easy by using what we know about parallelograms. We will begin by defining "base" and "height" for a triangle. Once again, we will allow complete freedom in picking any side as the base. But once the base has been chosen, the height will be the distance from the angle not touched by the base to the line on which the base lies. The following pictures of the same triangle show different choices for "base" and "height."

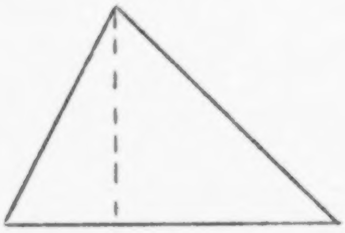


Sometimes it is convenient to be a bit fussy about which side is chosen as the base. This is especially true in the case of a "right triangle," one which has a right angle or square corner.

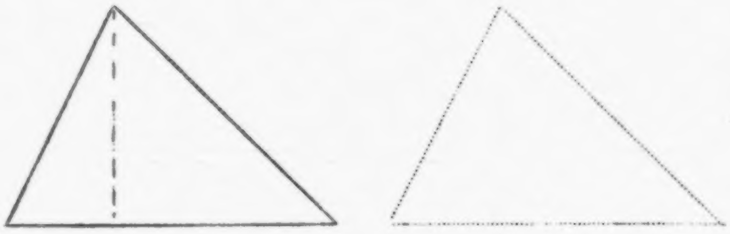




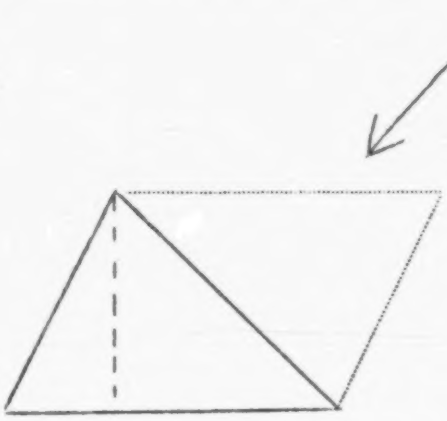
We are now ready to find a formula for the area of a triangle. Look at the triangle to the right.



Instead of molesting this triangle by cutting it apart, we will make an exact duplicate of the triangle.



Moving the new triangle into place forms a parallelogram with a base the same as the base of our original triangle and a height the same as the height of our original triangle.



We already know how to find the area of a parallelogram. We must multiply the base times the height. But the parallelogram is exactly twice too big. Therefore, if we want the area of the triangle, we must divide the area of the parallelogram by two. So it appears that the area of a triangle is given by the formula:

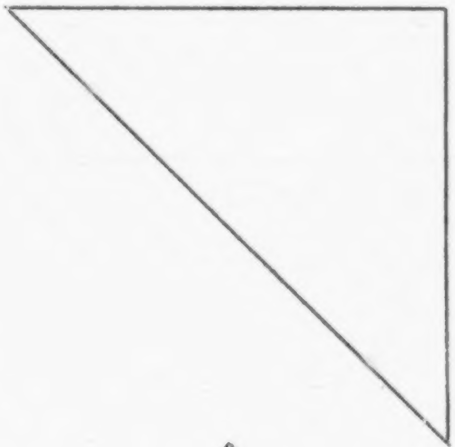
AREA = Base times Height divided by two

AREA = $\frac{b \times h}{2}$ or AREA = $\frac{1}{2}bh$

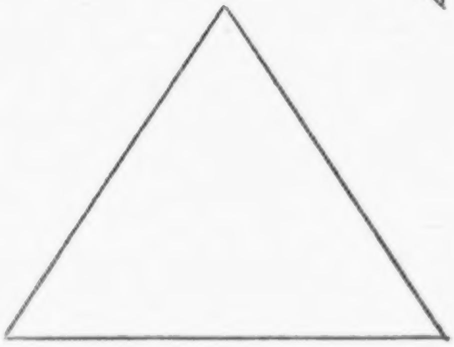
EXERCISES

1. Find the areas of the following triangles. (Use just a ruler. If you desire better answers by using tools to construct lines perpendicular to the base, you may do so. In most of the examples your eyes will be accurate enough. If your answer is very close to what the teacher gives, you may consider your answer correct.)

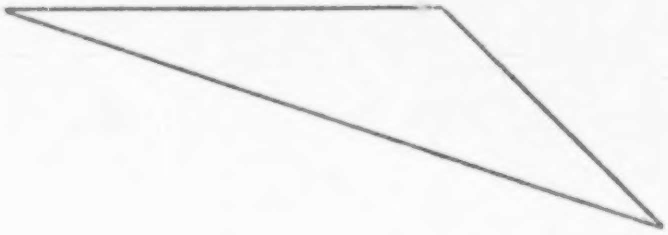
(a)



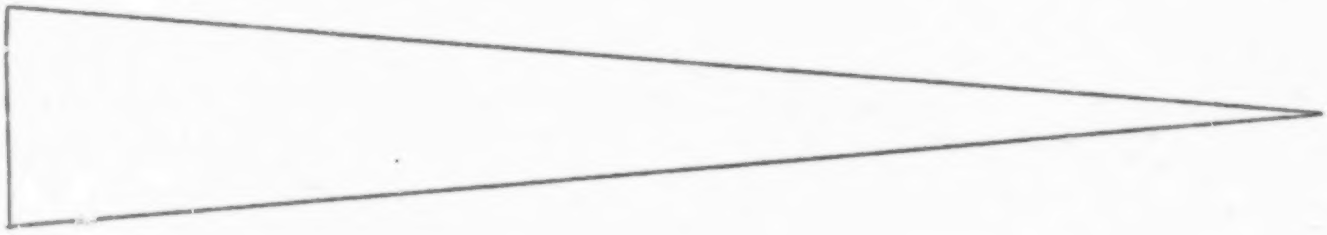
(b)



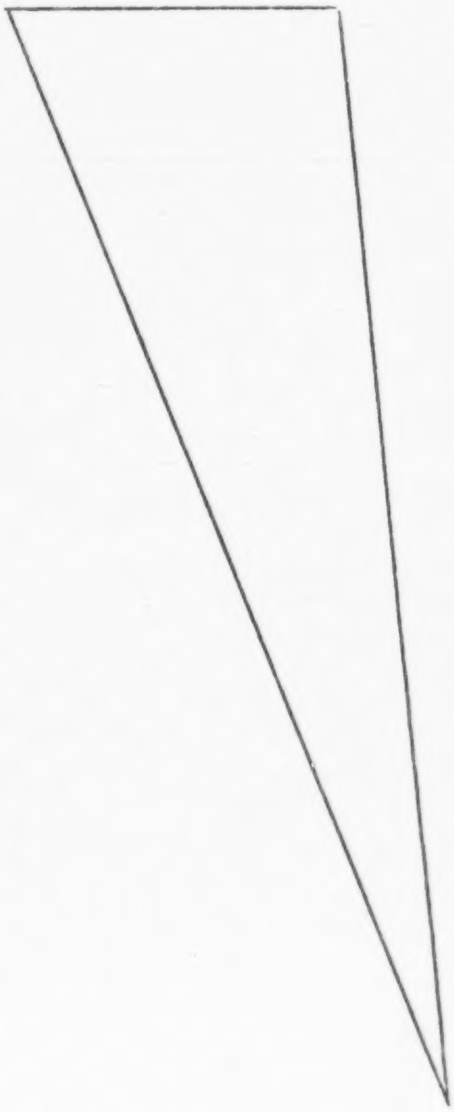
(c)



(d)



(e)



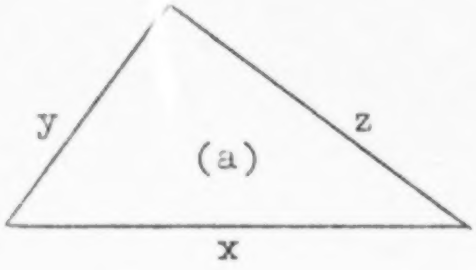
2. The sides of a triangle are 3 inches, 4 inches, and 5 inches. What is the area of the triangle? (You will have to draw the triangle first.)

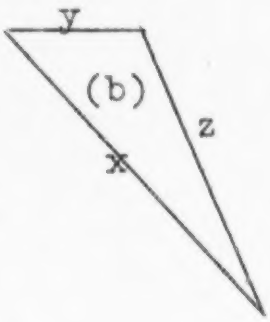
3. A triangle is 5 feet long and 3 feet high. What is its area?

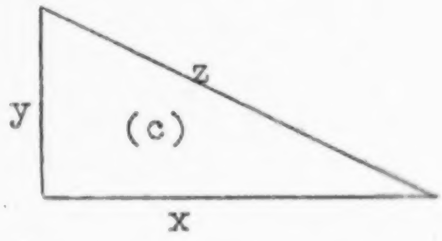
4. The area of a triangle is 25 square feet. The base is 5 feet. How high is the triangle?

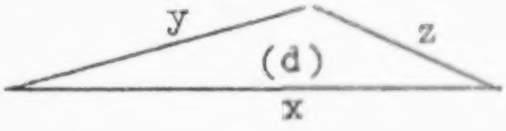
5. CLASS DISCUSSION How many different triangles are there with area 25 square feet and base 5 feet?

6. CLASS DISCUSSION If you had no tools for constructing perpendicular lines to given lines, which choices would you make for base and height in each of the following examples? (Note: If the entire class disagrees with you, don't let it bother you. Perhaps your eyes work differently than theirs.)

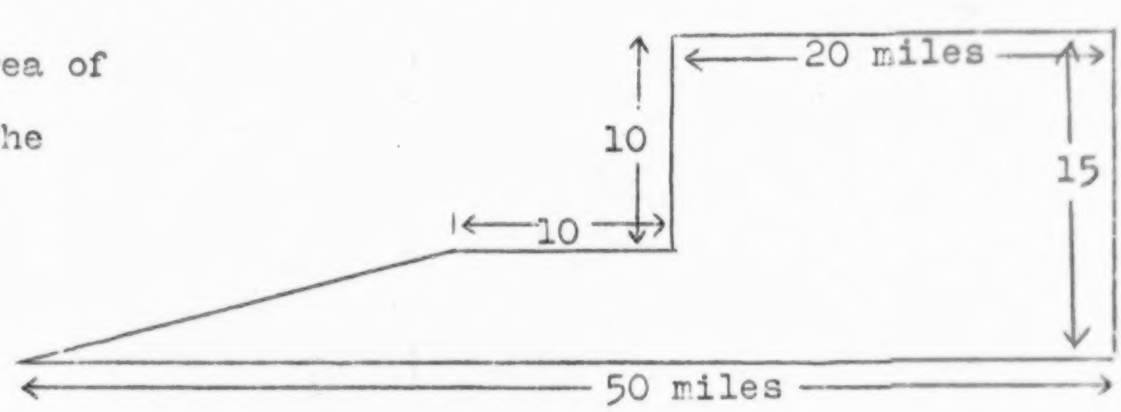








7. Find the area of the figure to the right.

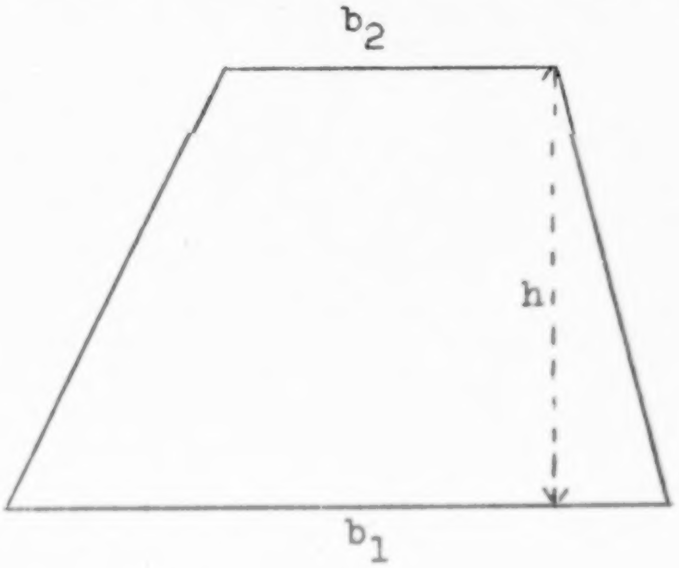


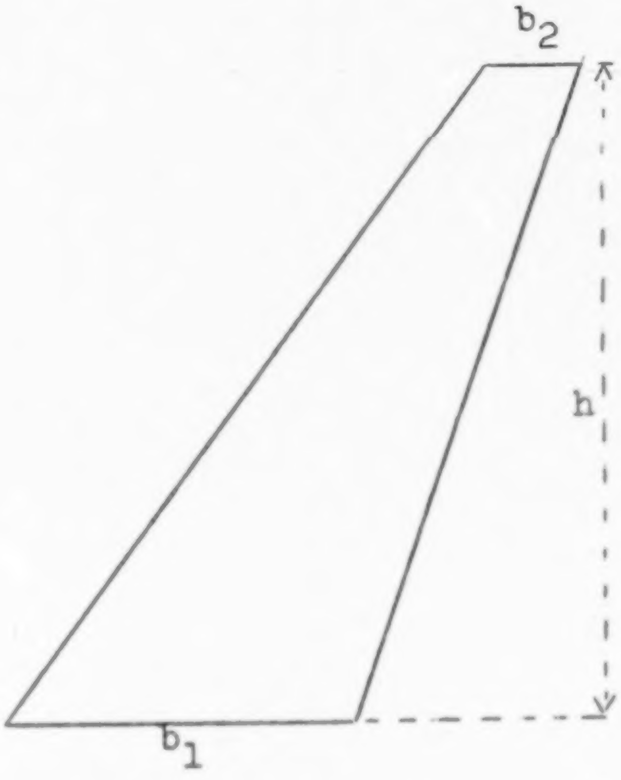
PART E

Area of a Trapezoid

The trapezoid is, at first glance, a somewhat peculiar-looking figure. It may be thought of as a rectangle which has been involved in an accident, this time in a rather terrifying accident. To define what we mean by a trapezoid, we would probably say something like this: A trapezoid is a four-sided figure with two parallel sides.

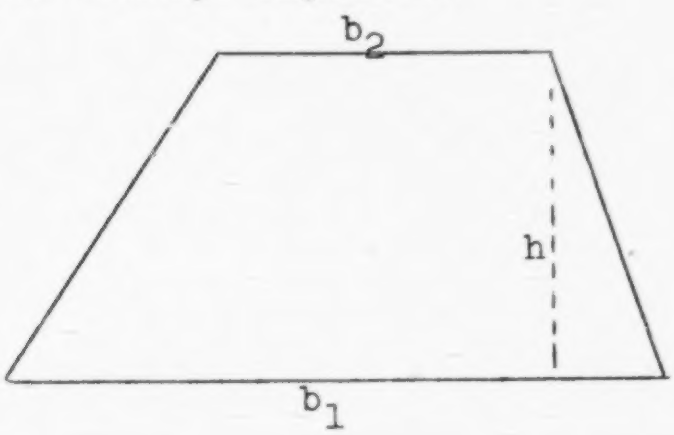
As we did with the triangle, we will begin by naming the parts of the trapezoid. We call the two parallel sides the first base and the second base, and we denote them with letters b_1 and b_2. The height, h, is the distance between the two bases. (Look at the following examples.)

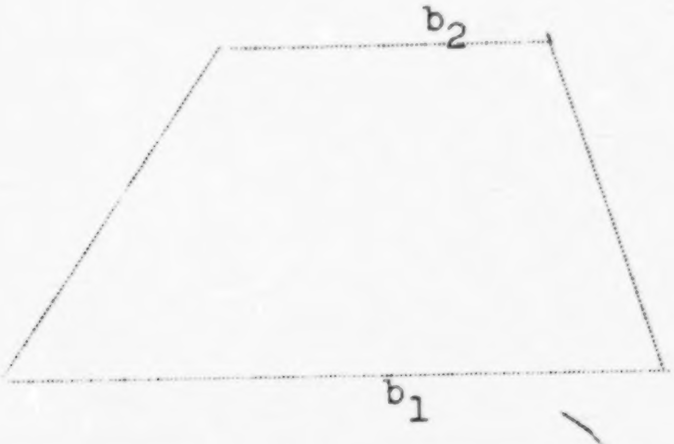




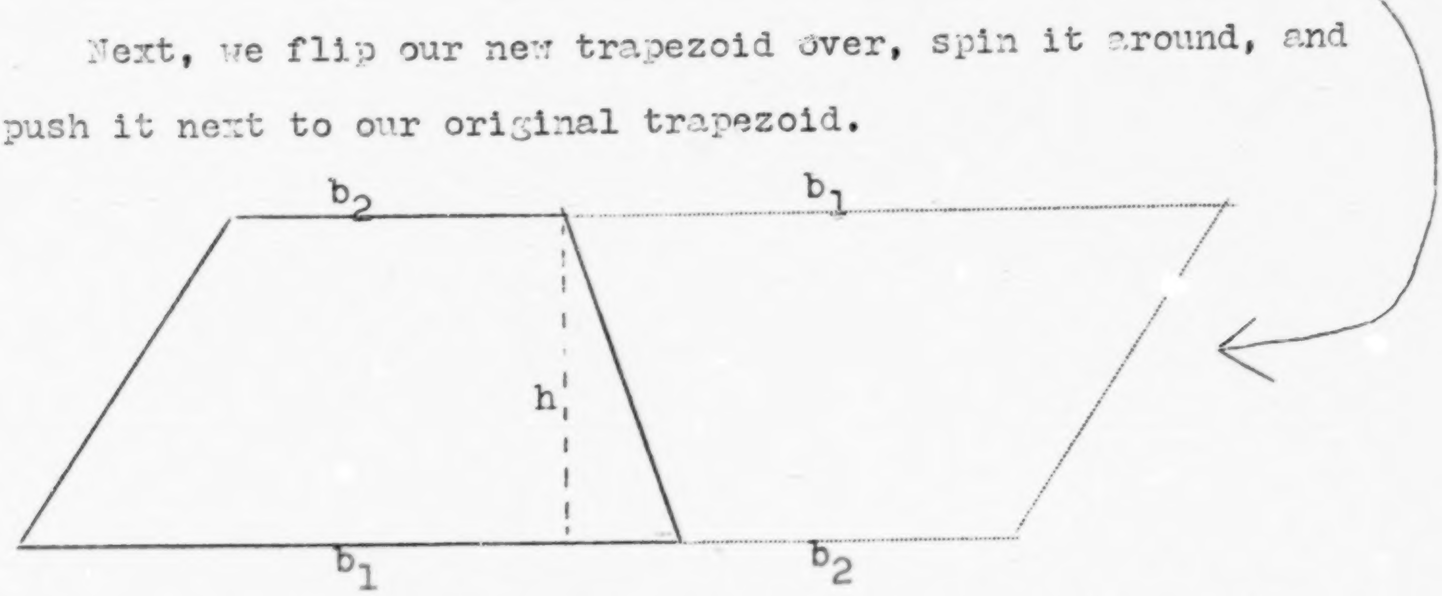
As we have been doing, we shall attempt to make the problem of finding the area of a trapezoid look like something familiar.

We do very much the same sort of thing that we did in the case of the triangle. We begin by making an identical twin for our lonely trapezoid.





Next, we flip our new trapezoid over, spin it around, and push it next to our original trapezoid.



The result is absolutely delightful. Once again we have a parallelogram. The length of its base is $b_1 + b_2$ and its height is simply h, the height of our original trapezoid. We know the area of the parallelogram, $(b_1 + b_2)$ times h. This area is twice too big since it is the area of two trapezoids which look exactly alike. To get the area of our trapezoid we must divide by two. This gives us the formula we want:

$$\text{AREA} = \frac{(b_1 + b_2) \times h}{2}$$

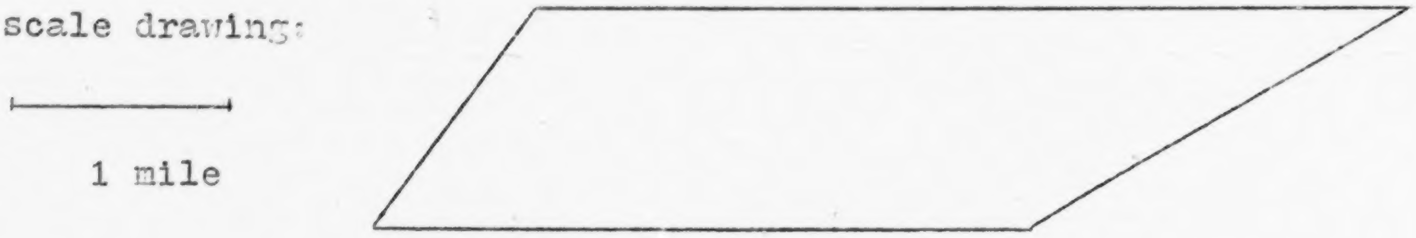
Once again, we were able to use what we already knew.

EXERCISES

1. Find the area of a parallelogram with bases measuring 6 inches and 10 inches and with height 5 inches.

2. Find the area of a triangle with a base of $3\frac{1}{2}$ feet and a height of 7 feet.

3. Find the area of the trapezoid pictured by the following scale drawing:



4. What is the area of a triangle with a base of 10 inches and a height of 6 inches?

5. If the bases of a trapezoid are 5 yards and 15 yards and the area of the trapezoid is 70 square yards, what is the height of the trapezoid?

6. If the area of a trapezoid is 48 square links, its base(the lower base) is 12 links, and its height is 8 links, what is the length of the other base?

NOTE: Don't feel sad if you can't make sense of this problem. Some of you may insist that we're not really talking about a trapezoid at all, and you are perhaps correct. The reason for giving this problem is to make it easier for you to discuss exercise 7.

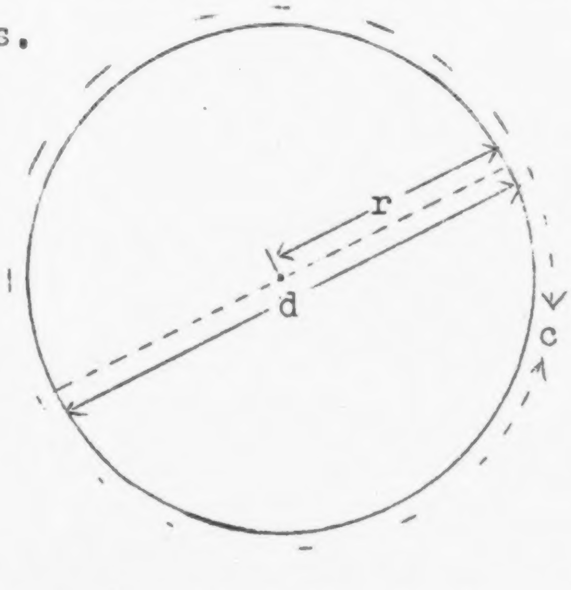
7. CLASS DISCUSSION If you know how to find the area of a trapezoid do you need to know any of the formulas which appeared before this?

PART F

Area of a circle

Finding the area of a circle presents a few more problems than the figures we have studied so far. It is not very easy to find the area of a circle by covering it with little square units of area. We would have to do a considerable amount of cutting and probably a good deal of guessing.

Before we try to cut the circle and make it look like something familiar, we shall name some of the parts of the circle and remind ourselves of a few relationships.



(a) The distance through the center of a circle is called the "diameter." We usually use the small letter "d" to mean "diameter".

(b) The distance from the center of a circle to the edge of the circle is called the "radius", and we usually use a small "r" to mean "radius".

(c) The distance around the circle is called the "circumference We usually use a small "c" to mean the "circumference

On the next page we will look at some important relationships which exist between the parts of the circle which we have just named.

Of "c," "d," and "r," the longest line is ______ and the shortest is ______.

If you are not familiar with one or more of the following rules, perhaps you, or your teacher, could perform a few measurements to convince yourself that the rules seem sound.

RULE (A) $d = 2r$

In other words, the diameter is twice as long as the radius.

RULE (B) $c = \pi d$

If you didn't already know, π is a number which is about $\frac{22}{7}$ or, if you prefer decimals, π is about 3.14.

Rule (B) says that the distance around the circle is always exactly π times d. This rule is a good one to know because it is usually easier to measure "d" than it is to measure "c".

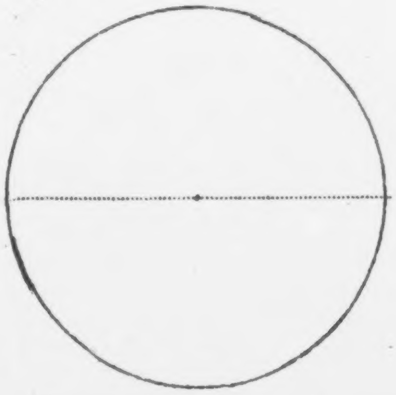
RULE (C) $c = 2\pi r$

If you agree that rule (B) is true, then you can easily see that rule (C) is also true by simply replacing "d" by "2r".

We will now begin to look for a method for finding the area of a circle. We will think of the circle as a pie. Our first task will be to cut the pie, much like you're used to seeing a pie cut.

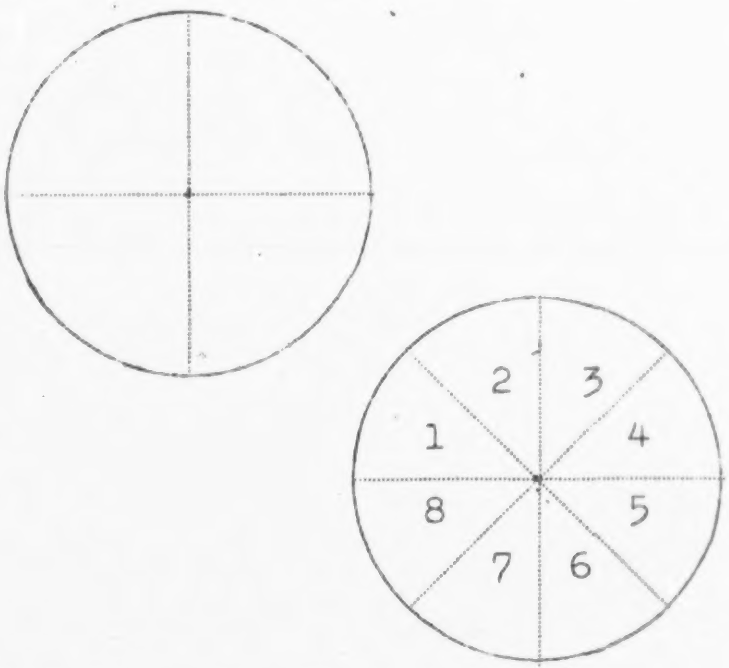
To make sure everyone does things the same, we will start by cutting the pie in half, making sure that the knife goes straight through the center of the pie.

The knife cut looks just like a diameter for our circle.

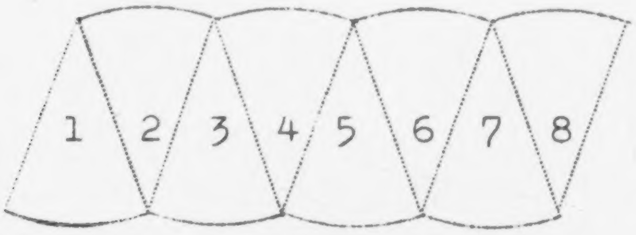


Next, we will cut the pie again, making four pieces of equal size. You will notice that each piece of pie has its sides equal to "r," the radius.

If we want to, we can cut a few more times so that each of the pieces we just had will be exactly cut in half. This will take two more diameters.

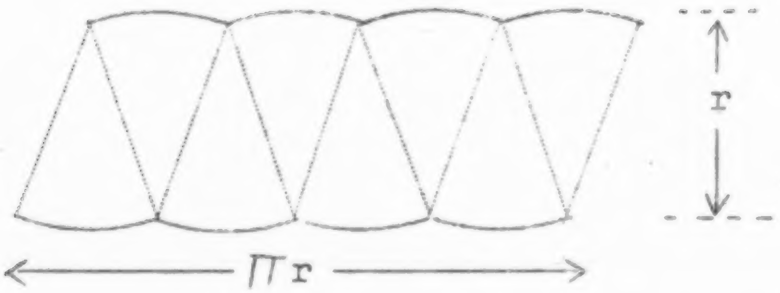


We are now ready to take the pieces of pie and try to put them together in such a way that will make the area of all the pieces look like something familiar.



You probably agree that the figure above looks like a badly bruised parallelogram. No matter how many times we would cut the pie, we would never get a perfect parallelogram, but cutting more often would leave smaller bumps on the top and bottom of what seems to look like a parallelogram. Several things would always be true, no matter how often we cut:

1. The height of our damaged parallelogram is roughly "r."
2. The length of the top edge added to the length of the bottom edge gives us πd. So the length of the base is $\frac{\pi d}{2}$ or πr.



We already know how to find the area of a parallelogram. We must take the length of the base times the height. In this case we must take πr times r, giving us πr^2.

We now have a formula for the area of a circle.

AREA = πr^2

You shouldn't feel surprised if you think that what was said about the area of the circle was sneaky and not at all accurate. Even though the pictures here are not very convincing, it does turn out that we have the correct formula. If you wish, you may cut a circle into many more than eight pieces to see that the more pieces we make, the smoother the top and bottom edges of the parallelogram become. In fact, the more pieces we use the more our parallelogram begins to look like a rectangle with length "πr" and width "r." If you doubt this, you should take a ruler and experiment with some circles. (You will very likely also want to use some tools to help make the pieces accurately.)

EXERCISES

1. Find the area of a circle with a radius of 4 centimeters.

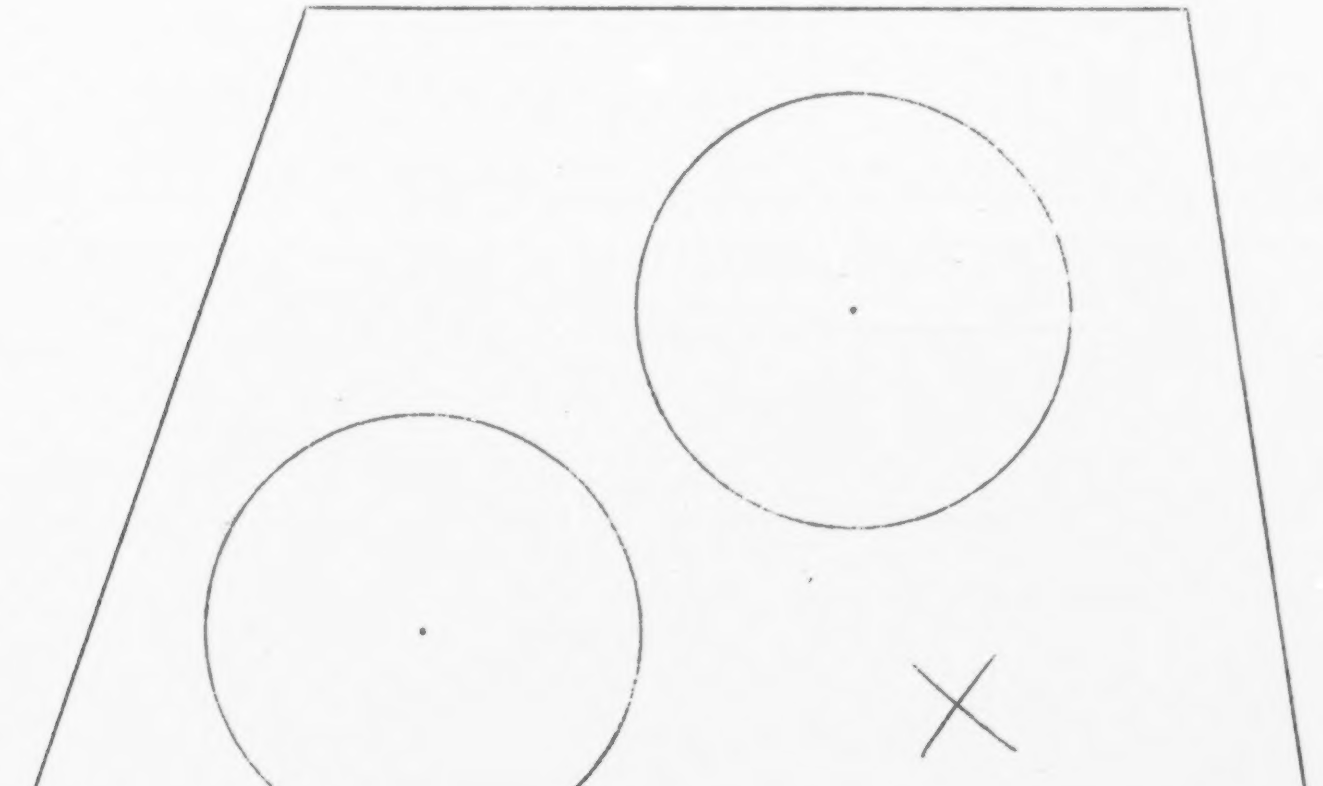
2. What is the area of a circle with a diameter of 22 yards?

3. If you are given the circumference of a circle, how would you find the area?

4. Find the area of a circle which has a radius of three feet.

5. The area of a circular table is 616 square inches. What is its diameter?

6. What is the area of the marked region in the following figure?

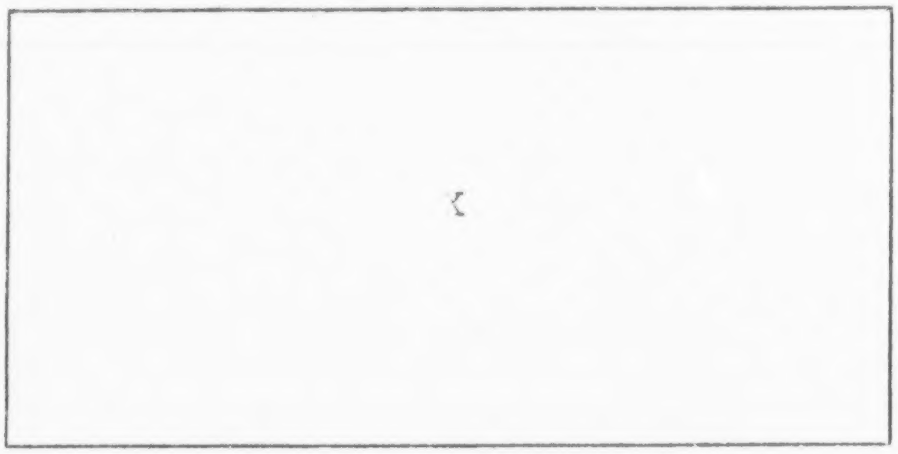


PART G

More exercises

Find the area of the marked region in each of the following:

1.



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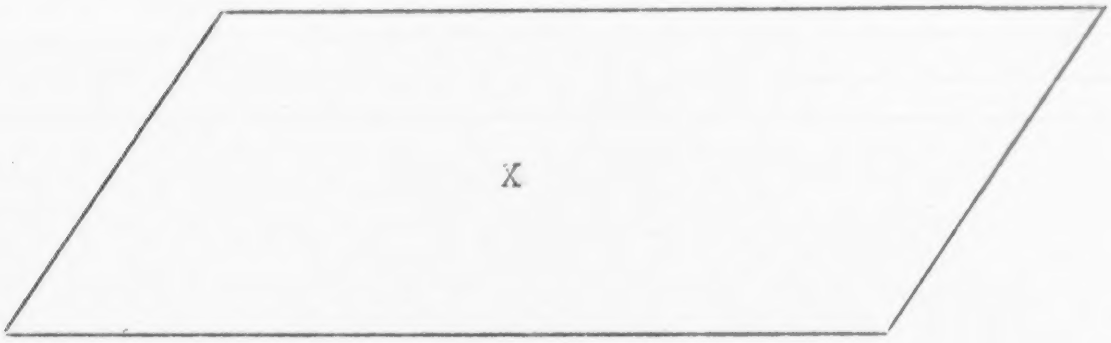

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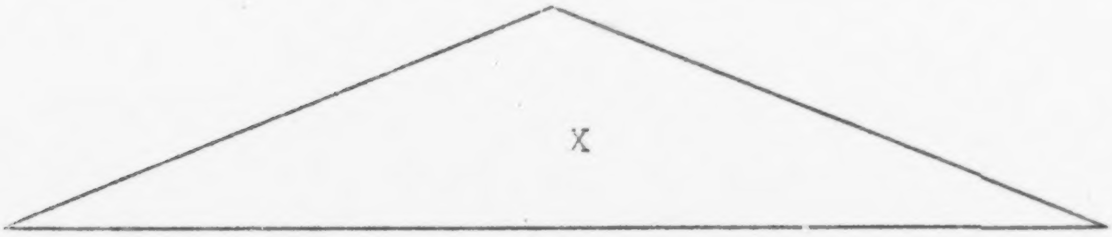

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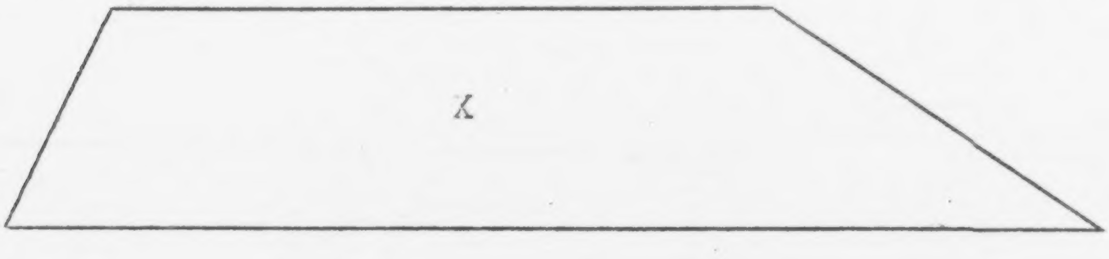
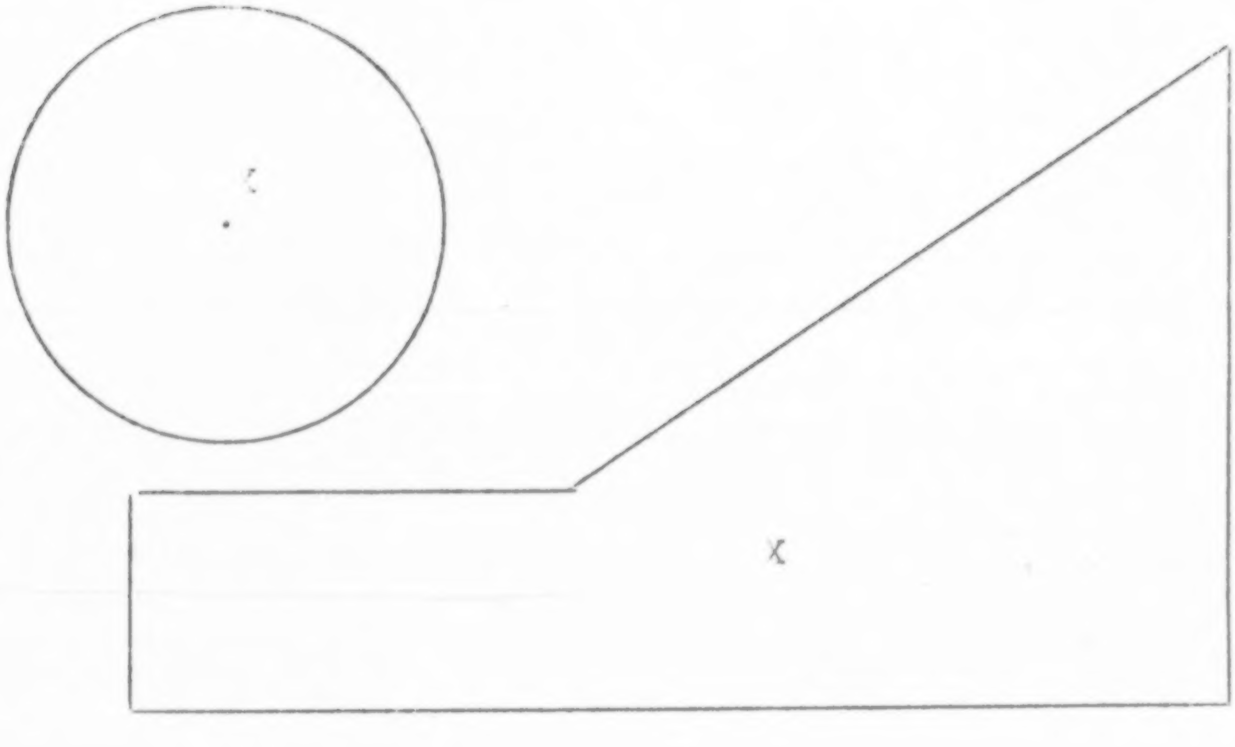


5.

6.



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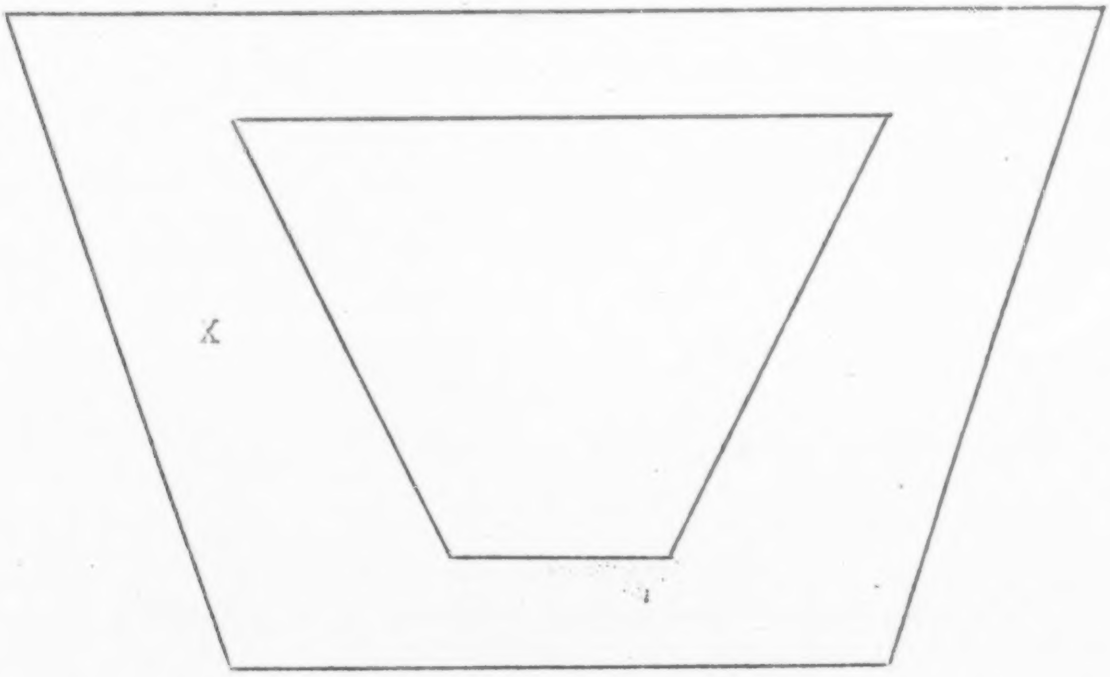

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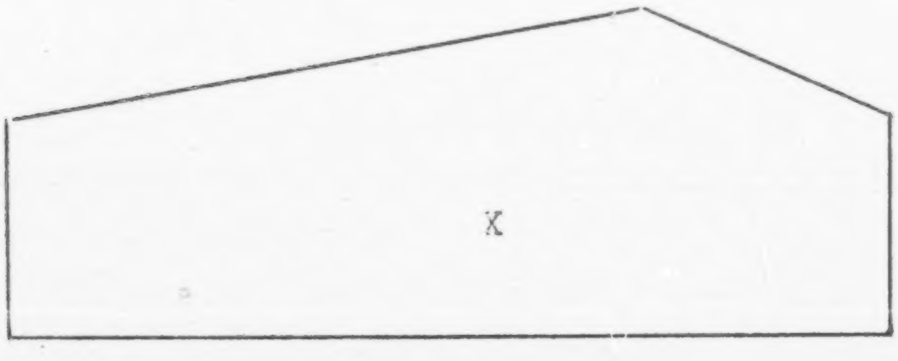

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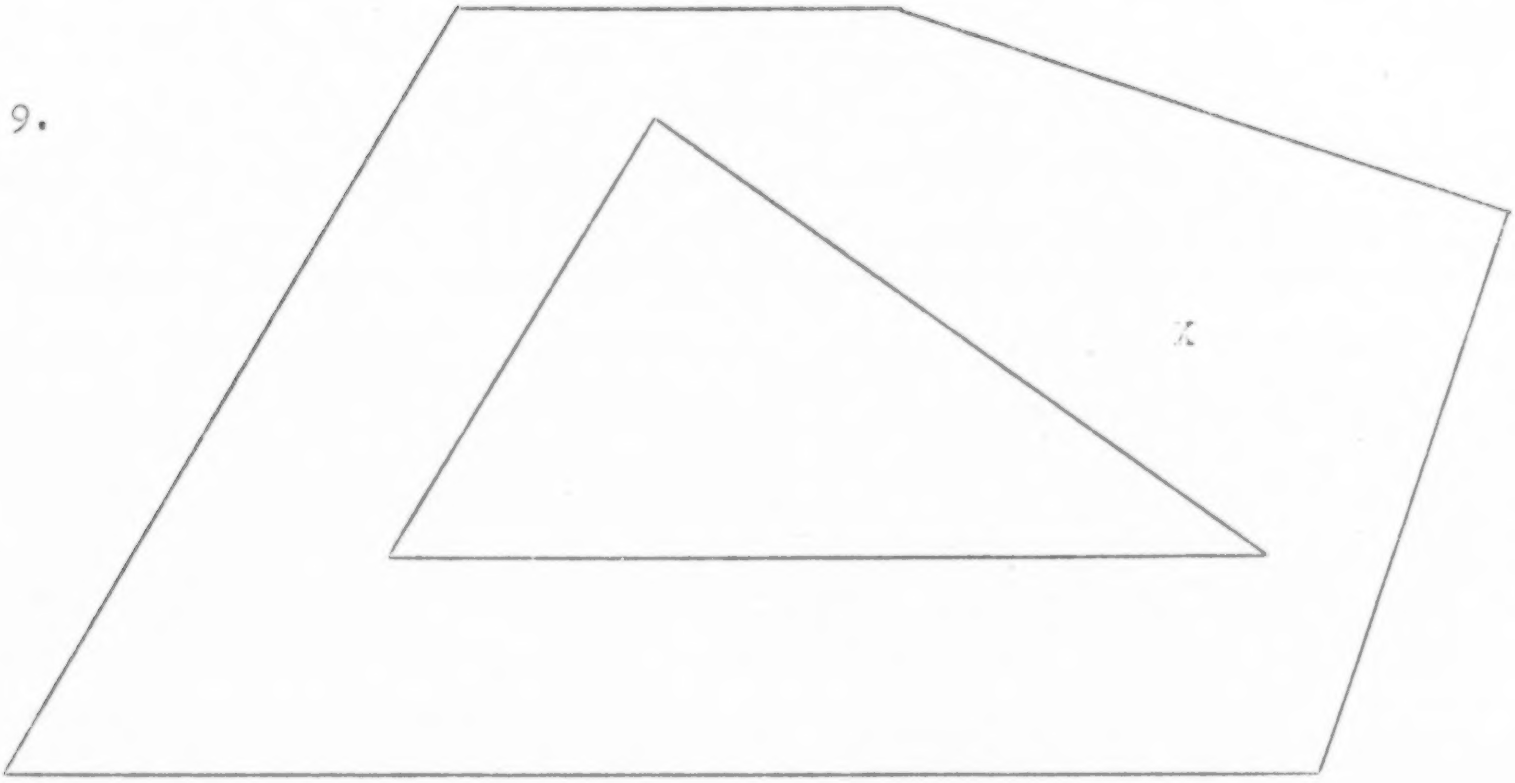
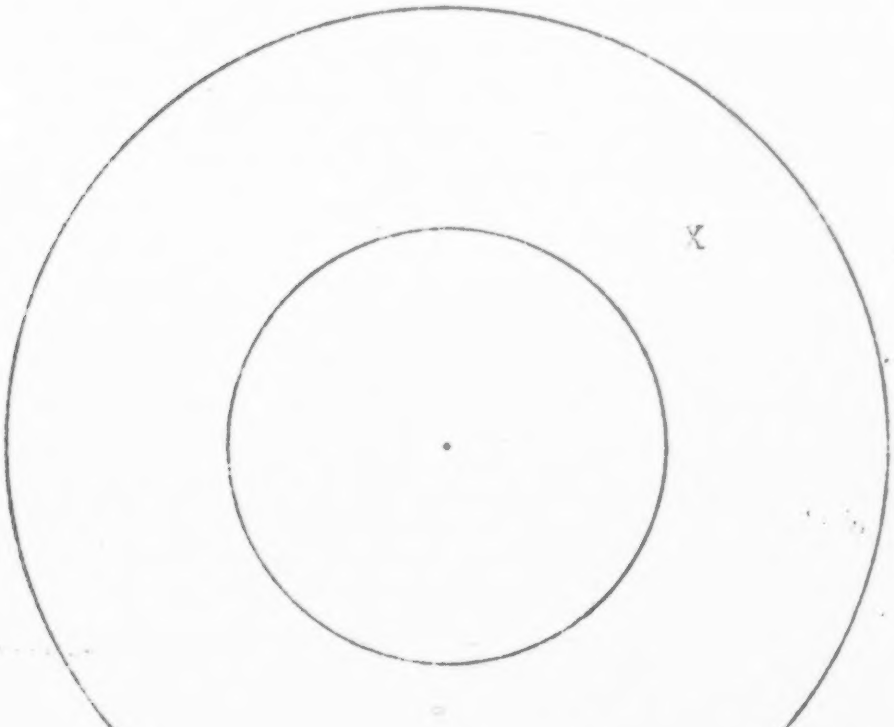


10.



11. Find the area of a triangle whose base is three thumbs long and whose height is four thumbs.

12. The circumference of the outside of a circular race track is 44 yards. The track is one yard wide. What is the area of the track?

13. The area of a trapezoid is 50 square furlongs, one base is 5 furlongs, and its height is 2 furlongs. What is the length of the other base?

14. Which of the following parallelograms are rectangles?

(a) base 6 inches, side 4 inches, area 24 square inches
(b) base 5 feet, side 6 feet, area 25 square feet
(c) base 3 miles, side 5 miles, area 15 square miles
(d) base 2 feet, side 3 feet, area 6 square inches

15. Which of the following trapezoids are parallelograms?

(a) first base 3 feet, height 2 feet, area 4 square feet
(b) first base 3 feet, height 2 feet, area 6 square feet
(c) first base 8 inches, height 10 inches, area 10 square ft.
(d) first base 2 feet, height 6 inches, area 1 square foot

16. A square has 4-foot sides. By joining the mid-points of each side with midpoints of sides not directly across the square we form a smaller square inside. What is the area of the smaller square?

17. A house with all rectangular rooms has 1,000 square feet of actual floor space. The kitchen has 130 sq. ft., both bedrooms are 120 sq. ft., and the bathroom is 80 sq. ft. The rather large living room is 25 feet long. How wide is the living room?

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Central Iowa Low Achiever Mathematics Project

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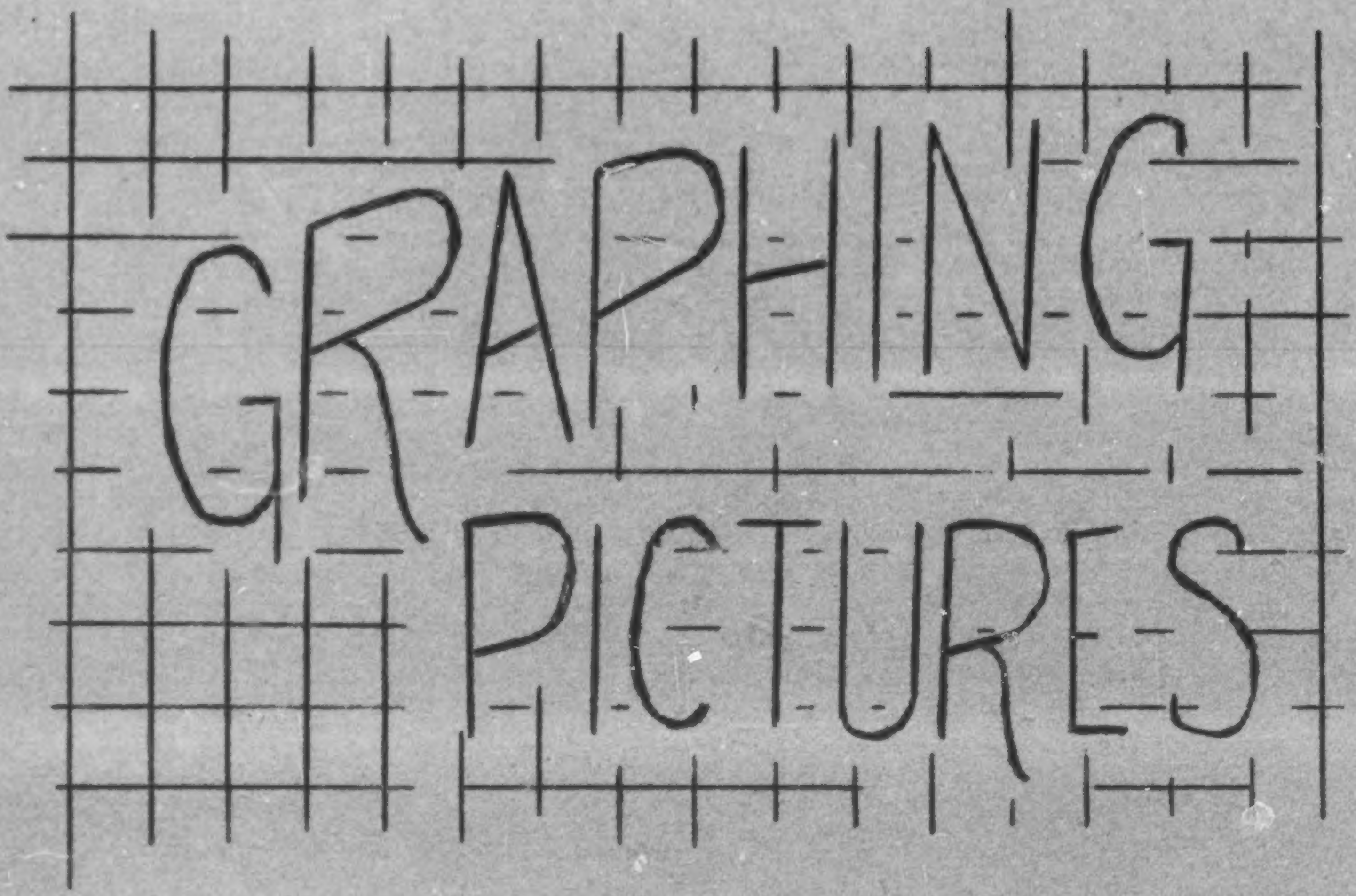

GRAPHING PICTURES

by

ROBERT C. MADISON

Consultants
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ACKNOWLEDGMENT

A special thank you is extended to Robert C. Madison's 9th grade algebra classes of the Des Moines Independent Community School District during the 1967-68 school year. Many of the pictures included in this booklet are their creations.

HOW TO USE THIS BOOKLET

STUDENT BACKGROUND

Each student should know how to plot (graph) points in a coordinate plane in all quadrants. How well he is able to do this is not important, as it is the intent of this booklet to provide the student with a fresh and enjoyable approach to strengthen his understanding of graphing. It is also suggested that the students be exposed to terms that apply to graphing (e.g., abscissa, ordinate, quadrant, etc.) They should also be able to recognize ordered pairs in table form or in the form (x,y).

FORMAT OF BOOKLET

Section A - contains five grid sheets (coordinate plane sheets) for graphing pictures. If the answer sheets are to be used for checking purposes then these grids must be used.

Section B - contains thirty direction sheets (two per page) for graphing pictures. It is suggested that the two sets of directions be separated by a dotted line so each set of directions may be used as an individual exercise. Direction sheets are arranged in order of difficulty (easy to difficult).

Section C - contains an answer sheet for all direction sheets. These may be removed from booklet and thermofax overlays made of each for ease in checking students' work.

ADDITIONAL SUGGESTIONS

A. Teacher-selected directions may be pulled from the booklet before the unit is started for the purpose of evaluation of students' progress.

B. After the first graph is completed, it may be desirable to provide different students with different directions. This will allow for individual differences and will provide a greater feeling of mystery as each student discovers the picture his directions reveal.

C. A sufficient number of direction sheets have been included so that the teacher may choose those exercises which best fit his or her class. It is not intended that every student complete every graph.

D. As students develop skills in graphing, they should be encouraged to make their own directions for a picture they have made. These graphs could then be added to the booklet for use with other classes or for further extension of this booklet.

E. A child's coloring book is an excellent source for additional ideas for pictures.

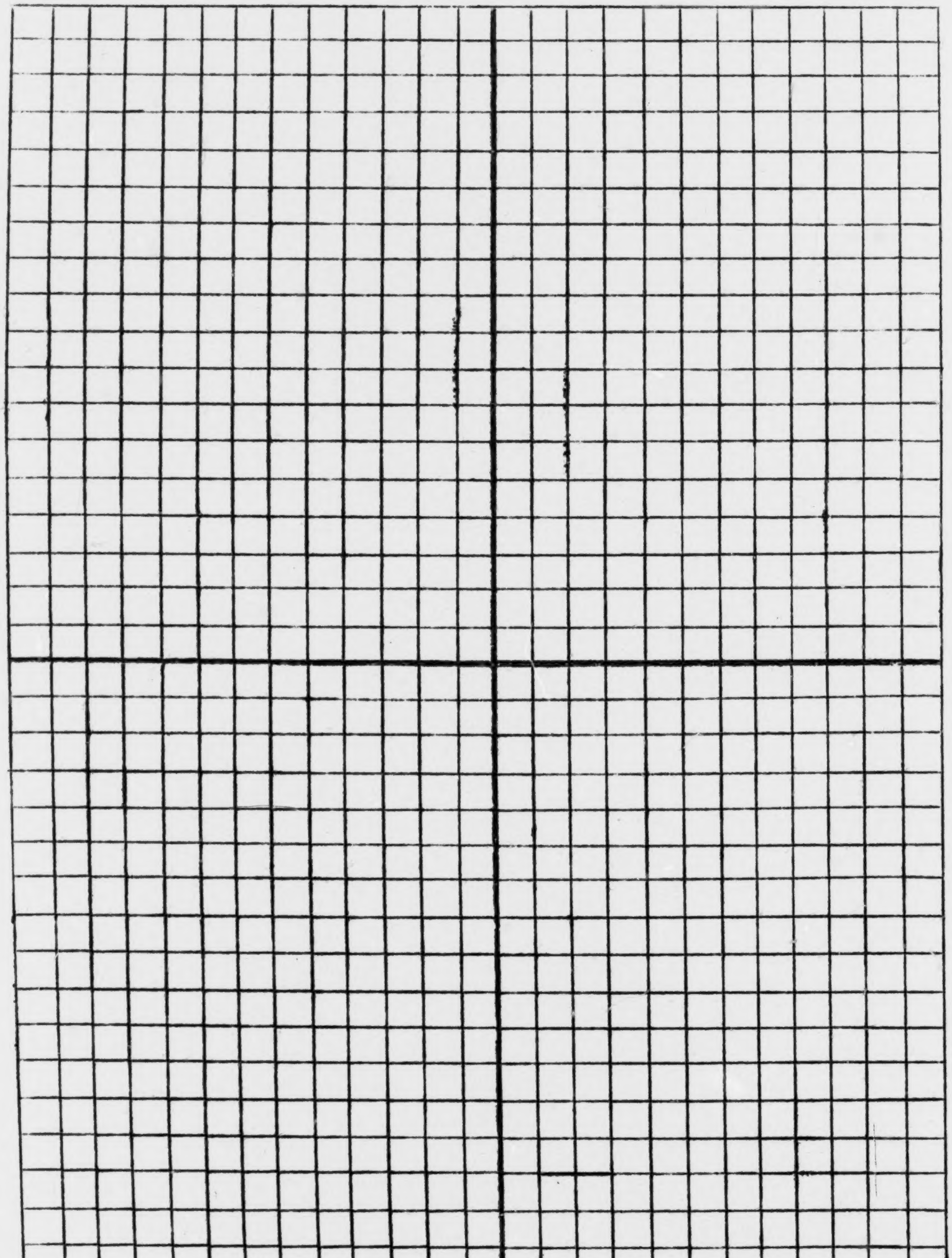
F. Necessary materials:
   1. Plenty of graph paper which has been made from the grid sheet in the booklet.
   2. A ruler for each student.
   3. Direction sheets.

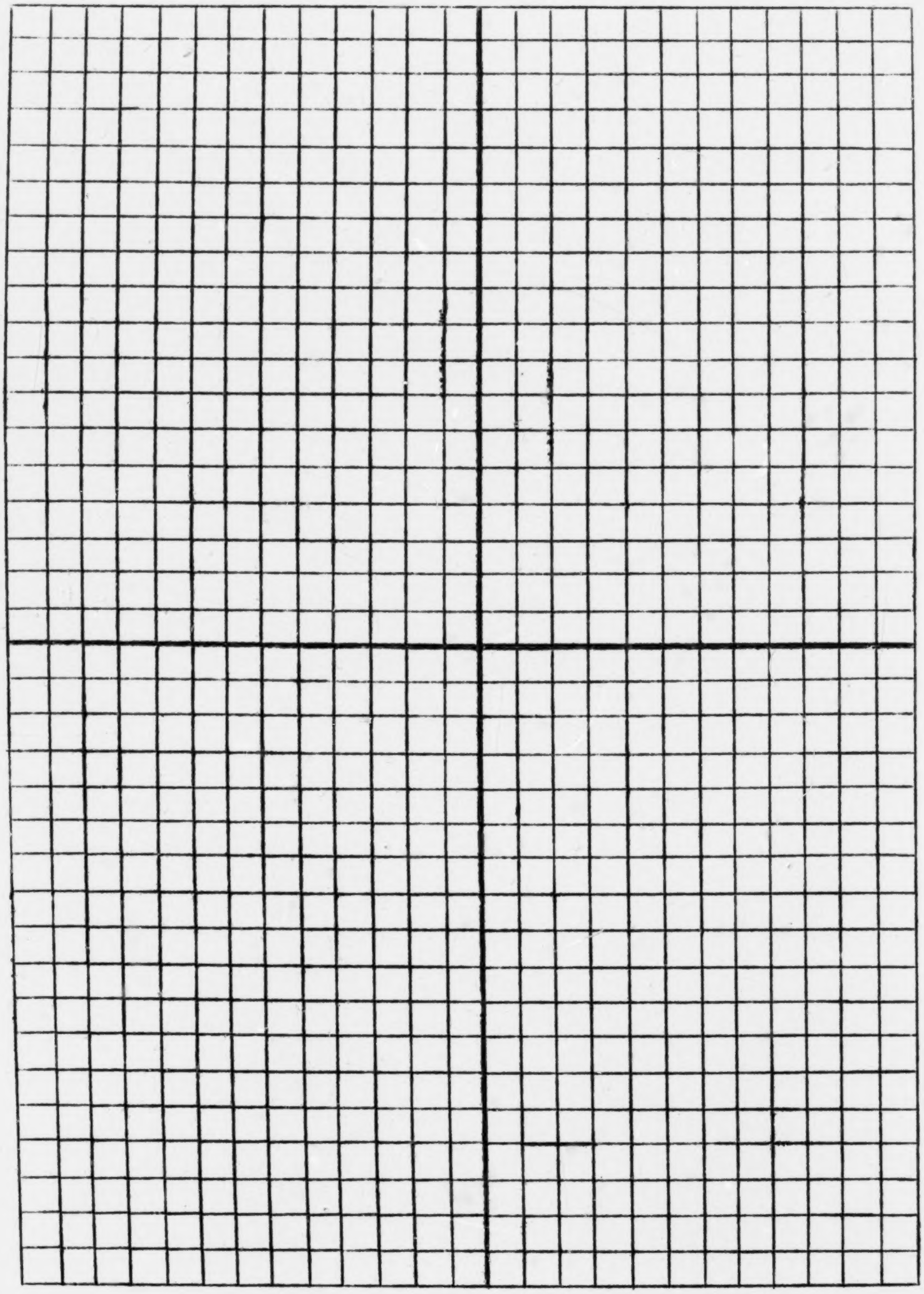
KEY TO PICTURES

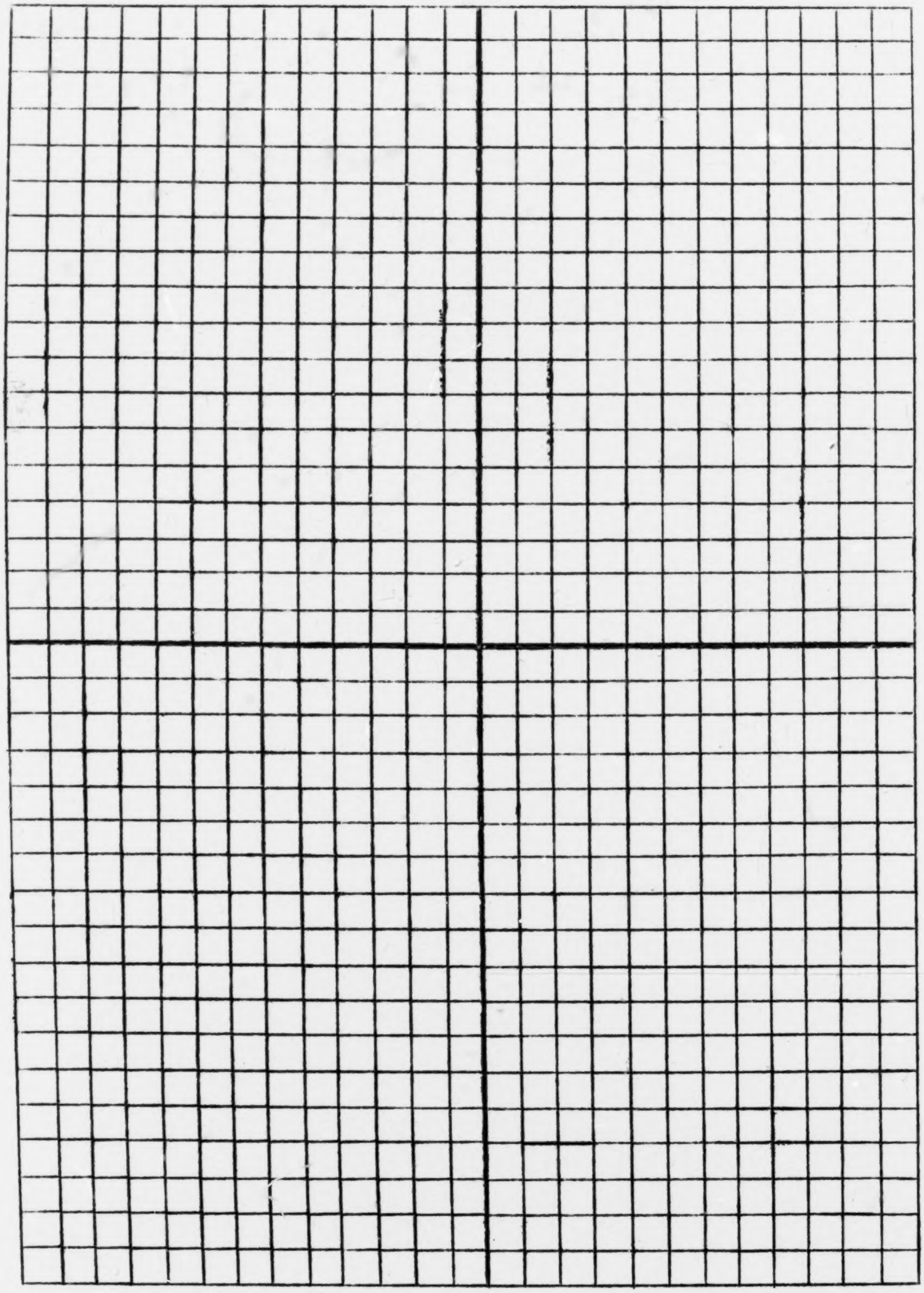
1	JET
2	BLOCKS
3	PEACE
4	LAMP
5	MICKEY MOUSE
6	TELEVISION SET
7	MASK
8	CANDLE
9	SAILBOAT
10	A E MILK BOTTLE
11	LANTERN
12	CAT
13	COFFEE POT
14	STOP SIGN
15	WATERING PAIL
16	PIG
17	FLYING FISH
18	TURTLE
19	GOLDFISH
20	DUCK
21	WINDMILL
22	PARROT
23	OWL
24	SEA HORSE
25	WALRUS
26	SEAGULL
27	ROOSTER
28	LION
29	DOG
30	GIRL

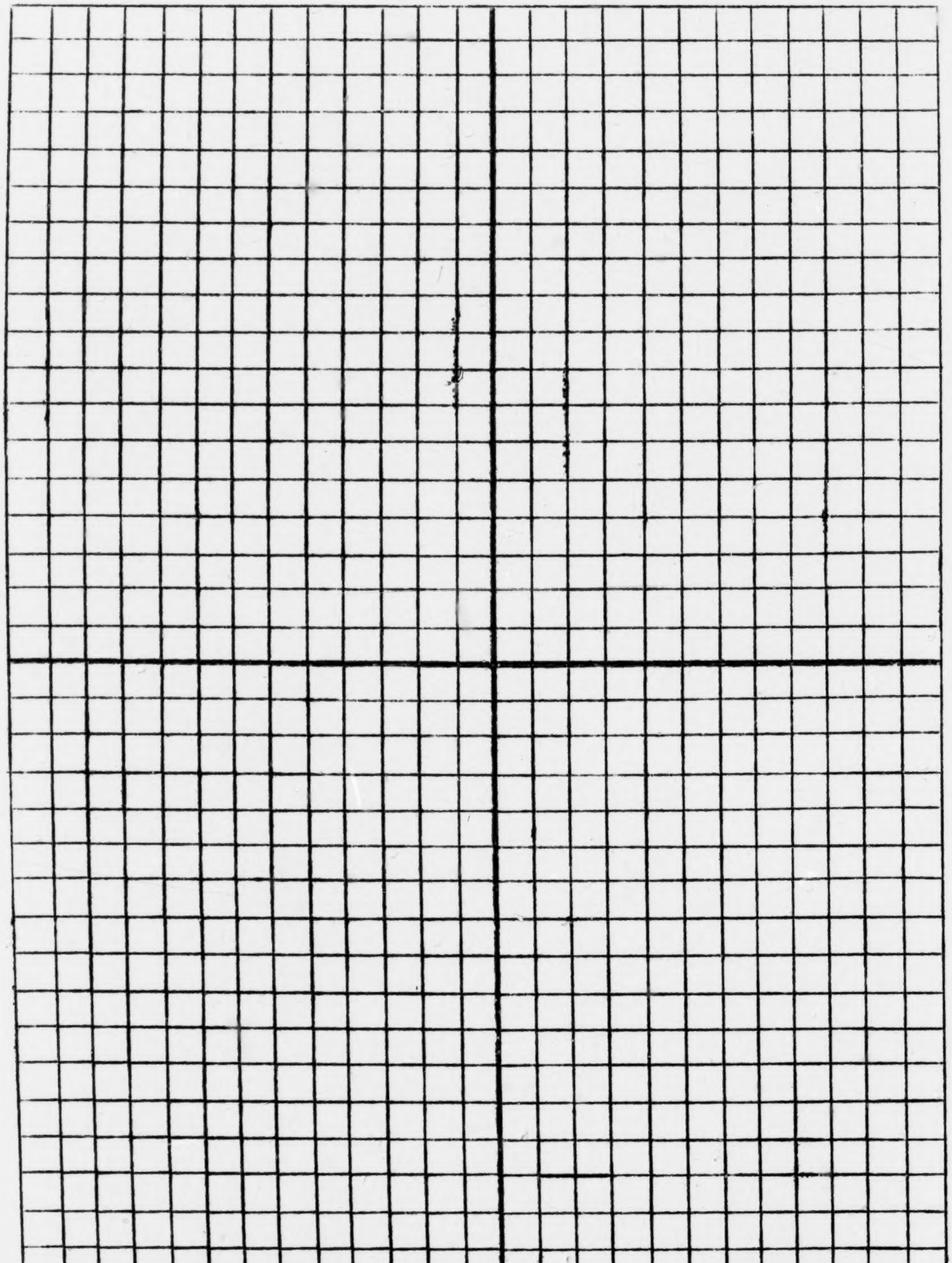
SECTION A

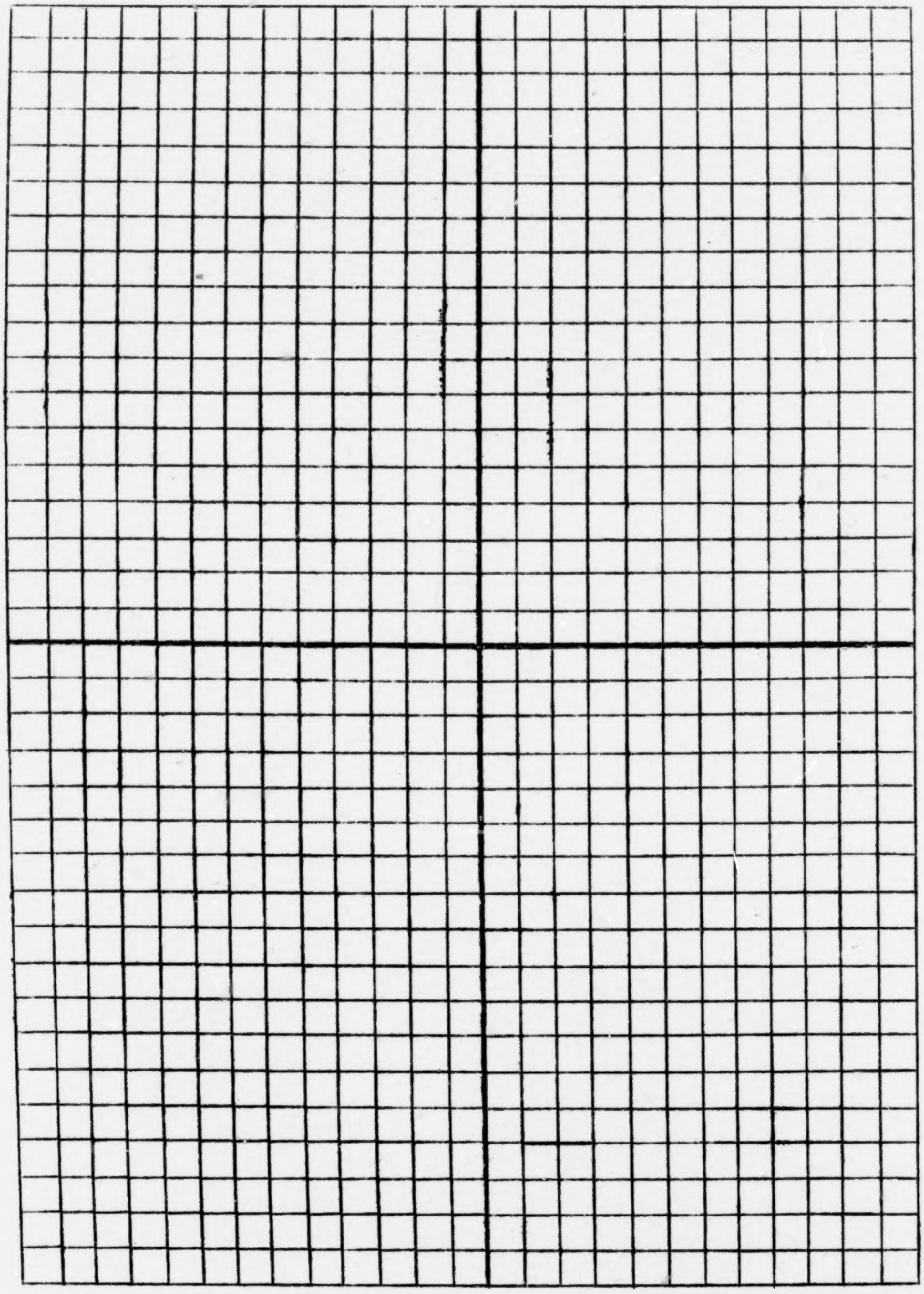
This section contains five grid sheets (coordinate plane sheets) for use in graphing the pictures in this booklet. These grids can be thermofaxed on a spirit master for reproduction of classroom sets. The answer sheets in Section C have been made from these grid sheets.











SECTION B

This section contains thirty direction sheets (two per page) for graphing pictures. It is suggested that the two sets of directions be separated for individualized exercises. These directions are arranged in order of difficulty (easy to difficult).

GRAPHING PICTURE #1

X	Y
0	14
-1	11
-1	6
-10	-2
-10	-4
-1	1
-1	-6
-3	-8
-3	-9
-1	-8

X	Y
1	-8
3	-9
3	-8
1	-6
1	1
10	-4
10	-2
1	6
1	11
0	14

GRAPHING PICTURE #2

X	Y
-3	3
3	3
3	-3
-3	-3
-6	0
-6	6
0	6
3	3
lift	pencil
-6	6
-3	3
-3	-3
lift	pencil
-3	11
-2	10
0	10
2	8
6	8
3	11
-3	11
-3	6
lift	pencil

X	Y
-2	10
-2	6
lift	pencil
0	10
0	6
lift	pencil
2	8
2	4
lift	pencil
6	8
6	-1
3	-1

GRAPHING PICTURE #3

X	Y
-10	-3
-10	3
-8	3
-7	2
-7	0
-8	-1
-10	-1
lift	pencil
10	3
7	3
7	-3
10	-3
lift	pencil
6	2
5	3
3½	3
2½	2
2½	-2
3½	-3
5	-3
6	-2
lift	pencil

X	Y
-3	3
-6	3
-6	-3
-3	-3
lift	pencil
1½	3
1½	1½
½	3
-1	3
-2	1½
-2	-3
lift	pencil
-2	-½
1½	-½
lift	pencil
9	-½
7	-½
lift	pencil
-6	-½
-4	-½

GRAPHING PICTURE #4

X	Y
0	5
8	5
6	17
-4	17
-6	5
0	5
0	2
-4	-2
-4	-6
-1	-12
-1	-15
-4	-18
6	-18
3	-15
3	-12
6	-6
6	-2
2	2
2	5
lift	pencil

X	Y
-4	-2
-3	-3
-2	-2
-1	-3
0	-2
1	-3
2	-2
3	-3
4	-2
5	-3
6	-2
lift	pencil
-4	-6
-3	-5
-2	-6
-1	-5
0	-6
1	-5
2	-6
3	-5
4	-6
5	-5
6	-6

GRAPHING PICTURE #5

X	Y
10	0
8	2
7	2
5	0
5	1
7	3
7	7
5	9
1	9
-1	7
-1	5
-4	5
-5	4
-6	2
-6	-½
-7	-1
-8	0
-8	1
-8½	2
-9½	2
-10	1
-10	0
-9	-1
-9	-4
-8	-5
-7	-6
-5	-6
-6	-7
-5	-9

X	Y
-3	-10
0	-9
3	-7
5	-5
7	-7
8	-7
10	-5
10	0
lift	pencil
-5	0
-5	1
-4½	2
-3½	2
-3	1
-3	-1
-4	-1
-4	0
* -5	0
-5	-1
-4	-1

*SHADE IN
THIS AREA

GRAPHING PICTURE #6

X	Y
-10	8
12	8
12	-8
-10	-8
-10	8
lift	pencil
4	-6
-9	-6
-9	6
4	6
4	-6
lift	pencil
-6½	-8
-8	-11
-7	-10
-6	-8
lift	pencil
8½	-8
10	-11
9	-10
8	-8
lift	pencil
2	8
-1	8
* 0	9
1	9
2	8
lift	pencil
-3	13
0	9
lift	pencil

X	Y
6	-5
5	-6
5	6
6	5
6	-5
9	-5
10	-6
10	6
9	5
9	-5
10	-6
5	-6
lift	pencil
5	6
10	6
lift	pencil
6	5
9	5
lift	pencil
7	3
8	3
7	1
8	-1
7	-3
8	-3
7	-1
8	1
7	3
lift	pencil
4	13
1	9

*SHADE IN
THIS AREA

GRAPHING PICTURE #7

X	Y
-7	4
-9	4
-12	6
-9	3
-7	3
-7	-5
-2	-10
2	-10
7	-5
7	3
9	3
12	6
9	4
7	4
lift pencil	
0	8
-2½	11
-5	8
-2½	10
0	8
lift pencil	
0	3
-1	1
-1	0
1	0
1	1
0	3
lift pencil	
-7	3
-7	12
-4	14
-1	14
0	13
1	14
4	14
7	12
7	3
lift pencil	
-2	5
2	-5
lift pencil	

	X	Y
	0	6
	1	7
	2	7
*	2	8
	3	8
	3	7
	4	7
	5	6
	4	5
	3	5
	3	4
	2	4
	2	5
	1	5
	0	6
	lift pencil	
	0	8
	2½	11
	5	8
	2½	10
	0	8
	lift pencil	
	0	6
	-1	7
	-2	7
	-2	8
	-3	8
	-3	7
*	-4	7
	-5	6
	-4	5
	-3	5
	-3	4
	-2	4
	-2	5
	-1	5
	0	6

*SHADE IN
THESE AREAS

GRAPHING PICTURE #8

	X	Y
	-1	8
	-2	10
	-1	12
*	0	10
	-1	8
	-3	8
	-3	-11
	-8	-11
	-7	-13
	5	-13
	6	-11
	1	-11
	1	8
	-1	8
	lift pencil	

X	Y
5	-11
6	-9½
8	-9
8	-11
5½	-13
lift pencil	
4	-11
5	-9
7	-8
9	-8
9	-11½
5	-13

*SHADE IN
THIS AREA

GRAPHING PICTURE #9

X	Y
7	-1
-6	-1
-8	2
10	2
7	-1
lift	pencil
1	2
1	14
-8	4
1	4
lift	pencil
-7	1
-7	-1
-8	-1
-8	0
-7	0
lift	pencil

X	Y
1½	2
1½	4
5	4
1½	13
1½	14
1	14
1	13
1½	13
1½	4
1	4

GRAPHING PICTURE #10

X	Y
8	0
8	7
3	12
2	12
2	13
3	13
3	14
-3	14
-3	13
-2	13
-2	12
-3	12
-8	7
-8	-13
8	-13
8	0
lift	pencil
-5	2
-5	7
-4	9
-2	9
-1	7
-1	2
lift	pencil
-1	5
-5	5
lift	pencil
5	9
1	9
1	2
5	2
lift	pencil

X	Y
1	5½
4	5½
lift	pencil
-5	-5
-5	-1
-3½	-3
-2	-1
-2	-5
lift	pencil
-1	-1
-1	-5
lift	pencil
½	-1
½	-5
2	-5
lift	pencil
3	-1
3	-5
lift	pencil
5	-1
3	-3½
lift	pencil
3½	-3
5	-5

GRAPHING PICTURE #11

	X	Y
	2	11
	3	10
	3½	9
*	-3½	9
	-3	10
	-2	11
	2	11
	lift	pencil
	-3	9
	-3	4
	-5	-2
	-4	-5
	-3	-6
	3	-6
	3	-8
*	5	-12
	-5	-12
	-3	-8
	-3	-6
	-5	-6
	-7	-4
	-4	6
	-3	6
	lift	pencil
	-3	7
	-5	7
	-9	-4
	-6	-7
	-3	-7
	lift	pencil

X	Y
3	9
3	4
5	-2
4	-5
3	-6
5	-6
7	-4
4	6
3	6
lift	pencil
3	7
5	7
9	-4
6	-7
3	-7
lift	pencil
-1	-6
-1	-4
0	-4
-1	-3
-1	-2
0	1
0	3
1½	0
1½	-1
1	-2½
0	-4
1	-4
1	-6

*SHADE IN
THESE AREAS

GRAPHING PICTURE #12

X	Y
-1	9
3	9
6	11
5	9
6	6
3	2
5	0
9	-6
8	-7
3	-2
2	-3
0	-4
5	-10
3	-10
-2	-5
-4	-6
-6	-6
-7	-7
-9	-7
-8	-5
-12	-4
-8	-10
-10	-10
-17	-4
-17	-3
-15	-1
-16	0
-15	3
-14	4
-16	8
-11	11
-14	8

	X	Y
	-13	5
	-10	5
	-3	3
	-5	6
	-3	9
	-4	11
	-1	9
	lift	pencil
	2	3
	1½	2
	-½	2
	-1	3
	lift	pencil
	½	4
	1	3
*	0	3
	½	4
	lift	pencil
	1	5
	2	5
*	3	7
	1	5
	lift	pencil
	0	5
*	-1	5
	-2	7
	0	5

*SHADE IN
THESE AREAS

GRAPHING PICTURE #13

X	Y
4	7½
-4	7½
-5	7
-6	0
-6	-10
6	-10
6	0
5	7
4	7½
3	9
1	9
1	10
-1	10
-1	9
-3	9
-4	7½
lift	pencil
-5¼	5
-6	6
-7	6
-9	4
-9	0
-6	-4
lift	pencil

X	Y
-5½	3
-6	4
-7	4
-8	3
-8	0
-6	-2
lift	pencil
5½	3½
6½	4
7½	6
9	7
10	7
10	5½
9	5
8	2
7	0
6	-1

GRAPHING PICTURE #14

X	Y
-1	-3
-2½	-3
-5	-1
-5	3
-2½	5
1½	5
4	3
4	-1
1½	-3
0	-3
0	-11
* -1	-11
-1	-3
0	-3
lift	pencil
1½	0
2½	1½
1½	3
1½	-1
lift	pencil

X	Y
-1	3
-1	-1
lift	pencil
1	1
½	3
0	1
½	-1
1	1
lift	pencil
-3	0
-3	-1
-2	-1
-2	1
-3	1
-3	3
-2	3
-2	2
lift	pencil
-1½	3
-½	3

*SHADE IN
THIS AREA

GRAPHING PICTURE #15

X	Y
3½	0
9	5
11	5
9	8
8	6
3½	4
3½	-8
-7	-8
-7	8
3½	8
3½	4
lift	pencil
-7	-3
-10½	0
-12	3
-11	6

X	Y
-9	7
-7	6½
lift	pencil
-7	-2
-9	0
-11	3
-10	5
-9	6
-7	5½
lift	pencil
-3	8
-2	9
-1½	10
-1	11½
2	10
3½	8

GRAPHING PICTURE #16

X	Y
-6	8½
-7½	6
-7½	4
-9	3
-9	1
-7½	1
-7	1½
lift	pencil
-8	1
-8½	0
-5	-3½
-3½	-3½
-3½	-2½
-3½	-5½
-½	-5½
-½	-2½
lift	pencil
-½	-3½
4½	-3½
4½	-2½
4½	-5½
7½	-5½
7½	-2½
lift	pencil

X	Y
7½	-3
9	-2½
10	-1
11	1
11	5
7½	9½
6	10
-1	10
-2	9½
-4	9½
-6	8½
lift	pencil
11	5
12½	3½
13	4
12½	5
11½	3½
12½	2

GRAPHING PICTURE #17

X	Y
-8	-1½
-6	0
-4	3
--2	4
0	4
3	3
5½	0
7	-5
5	-6
3	-8
2	-12
5	-11
7	-9
8	-11
11	-14
10½	-8
9	-5½
9	-4
10½	2
10½	5
9	8½
7	11
5	12
1½	13½
-1	13½
-4	13
-7	11
-9	9

	X	Y
	-11	6
	-12	2
	-11	-1
	-10	-2
	-9	-1
	-8	1
	-8	-1½
	lift	pencil
	-3	3½
	-1	1
	1½	-1
	½	1
	½	2
	1½	3½
	lift	pencil
	1½	13½
	3	15
	6	16
	8½	15½
	7½	15
	6	13
	5	12
	lift	pencil
	-9	5½
	-9½	5
*	-9	4½
	-8½	5
	-9	5½

*SHADE IN
THIS AREA

GRAPHING PICTURE #18

	X	Y
	8	-1
	7	-2
*	9	-2
	8	-1
	8	0
	4	4
	3	4½
	-5	4½
	-6	4
	-8	2
	-9	1½
	-8	0
	-9	-1½
	-8	-2
	-6	-2
	-5	-1
	-4	-1
	-3	--2
	-5	-4
	-6½	-4
	-7	-3
	-6	-2
	lift	pencil
	7	-2
	5	-2
	4	-1
	3	-1
	1	-3
	1½	-4
	3	-4½
	5	-2
	lift	pencil
	-3	-2
	-3	-3
*	-4	-3½
	-4½	-3½
	-3	-2
	2	-2
	lift	pencil
	5	-2
	5	-3
*	4	-3½
	3½	-3½
	5	-2
	lift	pencil

	X	Y
	-8¾	1
	-9	1
	-10	1½
	-11	1½
	-13	0
	-13	-1
	-11	-1¼
	-10	-1½
	-9	-1
	-8¾	-1
	lift	pencil
	-13	-1
	-11½	-1
	lift	pencil
	-11½	½
*	-11	1
	-10	½
	-11½	½

*SHADE IN
THESE AREAS

GRAPHING PICTURE #19

X	Y
-12	14
-11	1
-10½	-2
-10½	-4
-11	-7
-12	-11
-9	-8
-6	-2
-5	0
-3	2
-½	4
2½	5
5	5
7	4½
10	3
12	1
13	-1
12	-2
10	-2
9½	-1½
9½	-2
10	-2½
12	-3
13	-2½
12	-5
10	-7
7	-8½
5	-9
2	-9
0	-8½
-3	-7

X	Y
-6	-4
-9	1½
-11	3½
-12	4
lift	pencil
7	4½
5	9
2	12
-3	15
-6	15
-3	12
-2½	10
-2	4
-1	3½
lift	pencil
5	-9
3	-14
0	-17
-5	-18
-6	-18
-4	-16
-2½	-13
-2½	-10½
-3	-8
-2	-7½
lift	pencil
8	0
9	1
* 10	0
9	-1
8	0

*SHADE IN
THIS AREA

GRAPHING PICTURE #20

X	Y
0	2
-3½	0
-4	2
-3½	4
-2½	6
-1	8
-½	10
-1	11½
-2	13
-4	13
-6	12
-7	10½
-6½	9
-10½	7½
-9	7
-6	8
-6½	9
lift	pencil
-9	7
-5½	7
-6	8
lift	pencil
-5½	7
-9	1
-9½	-1
-9	3
-7	-6½
lift	pencil
-10	-6
-8	-7

X	Y
-6	-6
-4	-7
-2	-6
0	-7
2	-6
4	-7
6	-6
8	-7
10	-6
7	-6½
8	-7
9	-6½
10½	-7
lift	pencil
7	-6½
9	-5
11	-3
10	-2½
11	-1½
9½	-2
10	0
8½	-1
4	2
0	2
lift	pencil
-2	-4
0	-5½
2	-5½
4	-5
4½	-4
3½	-4
4	-3½
3	-3½
3½	3
2½	3
lift	pencil
-5	10
* -4½	10
-5	10

*SHADE IN
THIS AREA

GRAPHING PICTURE #21

X	Y
½	3
½	-3
lift	pencil
2½	2½
-1½	-2
lift	pencil
-1½	2
2½	-2
lift	pencil
3½	0
-2	0
-3	1
-12	3
-12	-3
-3	-1
-2	0
lift	pencil
½	3
0	3
-1½	8
2½	8
1	3
½	3
lift	pencil
2½	2½
2	3
4½	7½
7½	4½
3	2
2½	2½
lift	pencil
3½	0
3½	½
9	2
9	-2
3½	-½
3½	0
lift	pencil
2½	-2
3	-1½
7½	-3½
5	-6½
2	-2½
2½	-2
lift	pencil

X	Y
1	-3
2½	-8
-1½	-8
0	-3
1	-3
lift	pencil
-1½	-2
-1	-2½
-3½	-7
-6	-4½
-2	-1½
-1½	-2
lift	pencil
-1½	2
-2	1½
-6½	4
-3½	7
-1	2½
-1½	2
lift	pencil
-1	-8
-4	-16
lift	pencil
2	-8
5	-16
lift	pencil
½	-10
-2	-16
lift	pencil
½	-10
3	-16
lift	pencil
-1	-14
2	-14
lift	pencil
-½	-12½
1½	-12½

GRAPHING PICTURE #22

X	Y
-4	8
-3½	7
-3½	3
-3	1
-2	0
0	-1
2	-1½
4	-1½
1	-17
3	-16
4	-17½
6	-1
6½	0
6½	1
10	0
9	1½
9½	2
8	2½
8½	3
7	4
7½	4½
5½	5½
6	6
3½	7½
-1	12
-3	13
-4	13¼
-5	13
-6	12
-6½	11
-6	10
-5	9
-4	8½
-4	8
lift	pencil
3½	7½
2	8
0	7½
-1	6

X	Y
-1	5
0	4
3	2
6½	1
lift	pencil
-6	10
-7	9½
-8	9
-8	7
-7	6
-6	6
-6½	7
-6	8
-5	8½
-5½	7½
-6	7
-5	7
-4	8
-5	9
-5½	10
-5	11
-4	11½
-3	11
-3	10
-3½	9
-4	8½
lift	pencil
-4½	10
* -4	10
-4	10½
-4½	10
lift	pencil
-2	0
-2	-1
-3	-1
-3	-2
-2	-3
0	-3
1	-2
0	-1

*SHADE IN
THIS AREA

GRAPHING PICTURE #23

	X	Y
	-7	-1
	-12	-1
*	-11	0
	-6	2½
	-7	-1
	-12	-10
	-10	-9
	-12	-12
	-10	-11
	-10	-14
	-8	-11
	-8	-13
	-7	-12
	-7	-14
	-3	-9
	lift	pencil
	2½	-2
*	8	-2
	2	2
	2½	-2
	2	-4
	-½	-8
	0	-9
	-1	-8
	-1½	-9
	-2	-8
	-2½	-9½
	-3½	-8½
	-4	-9½
	-4½	-8
	-5	-9
	-5½	-8
	-5	-5
	lift	pencil
	-12	-1
	-9	3
	-4	4
	-5	6
	-3½	10
	-4	13
	-2	10
	3	9
	7	11

	X	Y
	4½	8
	5	6
	4	4
	2½	2½
	6	1
	8	-2
	lift	pencil
	2	4
	1½	4½
*	2	5
	2½	4½
	2	4
	1	5
	1	6
	2	7
	3	6
	3	5
	2½	4½
	lift	pencil
	-2	5
*	-2½	5½
	-2	6
	-1½	5½
	-2	5
	-3	6
	-3	7
	-2	8
	-1	7
	-1	6
	-1½	5½
	lift	pencil
	0	5½
	-1	4½
	1	2
	½	5
	0	5½
	lift	pencil
	-1	4½
*	-½	3
	0	3¼
	-1	4½

*SHADE IN
THESE AREAS

GRAPHING PICTURE #24

X	Y
2½	12½
2	11
0	9
-4	8
-5	7
-5½	6
-7	8
-5	9
-4½	10½
-5	12
-4½	14
-3	15
-1	16
2	16
5	13
6	9
5½	4
4	0
4	-3
5	-5
7	-7
8	-9
8	-12
7	-14
5	-17
3	-18
1	-18
-1	-17
-1½	-15½
-1½	-14
0	-12
1	-12
2	-13
2	-14
1	-14
1	-13½
½	-13
-½	-14
-½	-16
1	-16½
3	-16
4	-15½
4	-13
3	-11
1	-10

	X	Y
	-1	-9
	-3	-8
	-4½	-5½
	-6	-3½
	-7½	0
	-6	2
	-4½	4
	-1½	6
	-1	7
	-1½	8½
	lift	pencil
	-6¾	6½
	-6½	7
	-5	8
	lift	pencil
	-2	11
	-2	12
	-3	12
	-3	11
	-2	11
	-4½	14
	-5½	15
	-4	15
	-4	16
	-3	16
	-3	17
*	-2	17
	-1	17½
	-1	17
	-½	16
	-1	16
	-3	15
	-4½	14
	lift	pencil
	5	3
	8	4
	7	2
	8	1
	6½	0
	7	-1
	6	-2
*	6½	-4
	5½	-5
	6	-6
	5	-5
	4	-3
	4	0
	5	3

*SHADE IN
THESE AREAS

GRAPHING PICTURE #25

X	Y
-5	-14½
1	-14½
lift	pencil
1	-8
1½	-10
½	-14
2	-15
6	-14½
9	-15
8	-13½
5	-12½
6½	-9
6½	-7
6	-5
lift	pencil
8½	-14
9½	-14½
10½	-9
10	-5
7	1
lift	pencil
6½	-1
7	2½
6½	5½
lift	pencil
5½	3½
6½	5½
6	8
4½	11
1	12½
-1	12½
-4½	11
-6	9
-7	6½
-7	5
lift	pencil
-7	6½
-8½	4½
-9½	0
-9	-5
-8	-8
-5	-10
lift	pencil

X	Y
-9¼	-2
-10	-5
-10	-8
-8	-13
-11	-14
-12	-14½
-9	-14½
-6	-15
-4½	-14
-6½	-12
-8	-8
lift	pencil
-1½	3½
½	1½
3	1½
4½	3
4½	5½
3	7½
-2	9
-5	8
-6	6
-5	3
-3½	2½
-1½	3½
lift	pencil
-3½	2½
-4	0
-4	-4
-5½	1
-5	3
lift	pencil
½	1½
0	-1
-2	-4
1	-2
2½	1½
lift	pencil
3½	8
3	8½
* 3½	9
4	8½
3½	8
lift	pencil

X	Y
-3	9
-2½	9½
-3	10
* -3½	9½
-3	9
lift	pencil
-3½	6½
-12	2½
lift	pencil
-3½	6
-12	½
lift	pencil
-3½	5
-12	-1½
lift	pencil
2	6
10½	0
lift	pencil
2	5½
10	-2
lift	pencil
2	5
9½	-3½

*SHADE IN
THESE AREAS

GRAPHING PICTURE #26

X	Y
-12½	1
0	6
1½	6
1½	8
2	9
4	11
7	11
9	9
6½	7½
9	7
12	7½
9	8
lift	pencil
7	7½
10	8½
12	8½
11½	9
9	9
lift	pencil
8½	7
6½	5½
5½	5½
lift	pencil
7	6
7	5
9	2
9	0
7	-3
3	-5
-5½	-5
-8	-3
-12	-5
-13	-5
-9	-2
-10	-1½
-11	-1½
-11	-½
-12	-½
-12	½
-12½	½
-12½	1
lift	pencil
-11	-1
-13	-1
-12	-2
-9	-2
lift	pencil

X	Y
-11	-2
-13	-3
-10½	-3
lift	pencil
4	4
6	3
7	2
7	0
4	-2
-1	-3½
-5	-3½
-8	-3
-9	-3
-9	-2
lift	pencil
4½	10
6½	10
lift	pencil
5	10
5	9
6	9
6	10
lift	pencil
-3	-5
-2	-6
-1	-6
0	-5
1	-6
2	-6
3	-5
lift	pencil
1	-6
1	-7½
½	-8
½	-9
1	-9½
1	-12
½	-13
5	-13
4½	-12½
2	-12½
2	-9½
2½	-9
2½	-8
2	-7½
2	-6
lift	pencil

X	Y
-2	-6
-2	-7½
-2½	-8
-2½	-9
-2	-9½
-2	-12
-2½	-13
½	-13
lift	pencil
-1	-6
-1	-7½
-½	-9
-1	-9½
-1	-12½
¾	-12½
lift	pencil
1½	6
4	5
4	4
0	2
-3	2

GRAPHING PICTURE #27

X	Y
-3½	11
-1½	10
-1½	13
0	15
-3	16
-4	19
-6	16
-8½	10
-11½	5
-12	0
-10	-5
-8½	-8
-5	-10
-2	-13
0	-12½
2½	-13
3½	-11½
3	-10
4½	-8
4½	-5½
5	-8
5½	-4½
6½	-8
7½	-5
7	-3
8½	-6½
9	-1
5½	-2
8	½
11	-1
9	4
5	½
7½	5½
11	5
9	7½

X	Y
7	8½
5½	5
4½	1
4½	5
6	9½
11	8½
8	11½
5	12
4½	11
3½	½
3	11
5	12½
10	11½
6	14
4	14
2	13
1	11
3	½
0	0
-3	2
-4	7
-3	14
-4	16
-6	16
-7½	18
-7	15½
-8½	14½
-6½	14½
-10	13
-11	11
-10	10
-8½	10
lift	pencil
-5	14½
-5½	15
* -5	15½
-4½	15
-5	14½
lift	pencil

X	Y
-5	-½
-6	-2
-4	-6
0	-8
2	-7½
-½	-6
3	-7
1	-5
4	-4
1½	-3½
2	-2
½	-3
lift	pencil
-2	-13
-1	-16
0	-16½
lift	pencil
-1	-16
-2	-17
lift	pencil
-1	-16
-3	-17
lift	pencil
-1	-16
-3½	-16½
lift	pencil
2½	-13
3	-16
4	-16½
lift	pencil
3	-16
2	-17
lift	pencil
3	-16
1	-17
lift	pencil
3	-16
½	-16

*SHADE IN
THIS AREA

GRAPHING PICTURE #28

X	Y
-2	4
* -3	5
-4	4
-2	4
-2	-2
2	-2
* 2	4
3	5
4	4
2	4
lift	pencil
-2	-2
* 2	-2
1½	-3
-1½	-3
-2	-2
lift	pencil
-4	-4
-2½	-5
2½	-5
4	-4
lift	pencil
-5½	-1½
-6	-2
-6	-½
-8	-1
-6	1½
-8	2
-6	3
-9	6
-6	5
-6	6
-4	4½
-5	8
-2	6
-2	10
0	6
2	10
2	6
5	8
4	4½
6	6
6	5
9	6
6	3
8	2
6	1½
8	-1

X	Y
6	-½
6	-2
5½	-1½
lift	pencil
-4	-1
-5	-2
-5	-5
-3	-7
-2	-7
-1	-8
1	-8
2	-7
3	-7
5	-5
5	-2
4	-1
lift	pencil
-5	-4
-9	-6
-7	-3
-9	-4
-9	-2½
-12	-2
-11	1½
-12	2
-11	4½
-12	6
-11	6
-11½	8½
-11	9
-12	12
-9	11
-10½	15
-7	13
-7	14
-6	13
-6	15
-5	13
lift	pencil
6	10½
5½	13
4½	12
4	15
2½	13
2	14
½	13
0	13½
-½	13
-2	14
-2½	13
-4	15
-4½	12

X	Y
-5½	13
-6	10½
lift	pencil
5	13
6	15
6	13
7	14
7	13
10½	15
9	11
12	12
11	9
11½	8½
11	6
12	6
11	4½
12	2
11	1½
12	-2
9	-2½
11	-4
7	-3
9	-6
5	-4

*SHADE IN
THESE AREAS

GRAPHING PICTURE #29

X	Y
-1	12
1	11
3	12
6	11
9	8
9	5
7	2
9	3
11	6
10	3
11	3
8	1
9	-1
9	-3
10	-3
11	-5
10	-5
11	-5
10	-6
8	-6
8	-5
8	-6
7	-5
7	-4
6	-4
6	-5
4	-6
2	-6
2	-5
4	-4
3	-3
2	-3
-1	-9
-4	-9
-5	-8
-6	-8
-5	-7
-6	-8
-7	-7
-6	-6
-7	-7
-8	-6
-8	-5
-7	-4
-8	-5
-9	-5
-9	-4
-7	-3
-3	-6
-4	-5¼

	X	Y
	-1	-2
	-2	-3
	-5	1
	1	1
	5	2
	3	3
	2½	4
	0	3
	0	2
	1	1
	0	2
*	-5	2
	-5	1
	-5	3
	-10	4
	-12	6
	-12	7
	-9	9
	-7	9
	-5	11
	-5	10
	-5	11
	-3	12
	-1	12
	lift	pencil
	-9	9
	-9	10
	-8	12
	-6	13
	-3	12
	lift	pencil
	1	11
	3	8
	2	9
	2	5
	2½	4
	lift	pencil
	3	-3
	5	-3
	6	-4
	lift	pencil
	-11	7
	-9	7
	-10	7
	-7	5
	-2½	5
	½	6½
	0	7
	1	6
	lift	pencil

	X	Y
	-3	10
	-2	11
	-1½	10
	lift	pencil
	½	-6
	3	-9
	5	-9
	6	-8
	6	-7
	5	-7
	5	-8
	5	-7
	4	-6
	4	-7
	4	-6
	2	-6
	1¼	-4½
	lift	pencil
	-5	6
	-7	6½
	-7½	7
*	-7	8
	-5	7
	-5	6
	lift	pencil
	-6	8
	-6	9
*	-5½	9
	-5½	8
	-6	8
	lift	pencil
	-3	7½
*	-3	8½
	-2½	8½
	-2½	7½
	-3	7½

*SHADE IN
THESE AREAS

GRAPHING PICTURE #30

X	Y
-2	-2
-3	0
-3½	2
-3½	4
-3	5
-2	6
0	7
1	7
3½	6½
5	5
3	8
5	6½
3½	9
4	9½
5	10
6	10
8	9
10	7
11	5
10½	8
9	10
7	12
lift	pencil
9	10
11	8
10	11
8	13
5	14½
0	15
-3	14½
-6	13
-5	15½
-6½	13
-7	15
-7	13
-9	15
-7½	12½
-10	13
-8	12
-10	10
-11½	6
-12	3
-12	2
-11½	-2
-10	-4

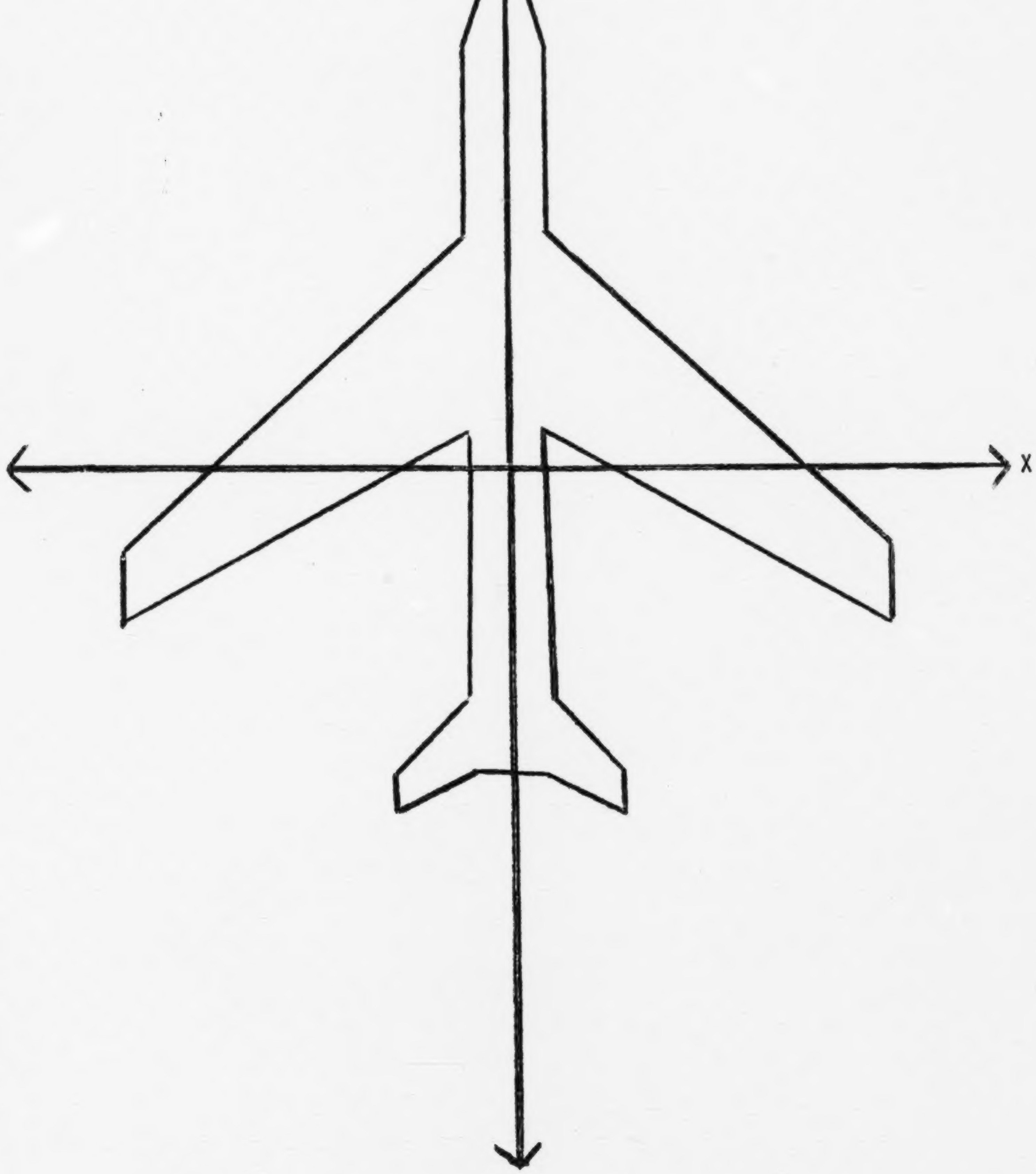
	X	Y
	-7	-6
	-5	-6½
	-1½	-5
	-3	-5
	-1	-4
	-2	-4
	0	-3
	-2	-2
	lift	pencil
	-5	-6½
	-4	-8
	lift	pencil
	-4½	-7½
	-4½	-8
	-3	-9
	0	-10
*	3½	-9
	1½	-11
	½	-13½
	-6	-9
	-4½	-7½
	lift	pencil
	8	9
	9	5½
	9	2½
	10	2½
	11	1½
	11½	0
	11	-2
	9½	-4
	8	-5
	6	-7
	5½	-8
	3½	-9
*	5	-10
	6	-11
	7	-13
	7½	-11
	6	-7
	lift	pencil
	0	3
	-½	4
	-½	5
	½	6
	2	5½
	3	4
	lift	pencil

	X	Y
	3	0
	3½	1
*	3	2
	2½	2
	2	1
	3	0
	lift	pencil
	5	0
	6	1
	7	1½
	8	1½
	8½	1
	8	0
	7	-1
	lift	pencil
	5½	5½
	5½	7
	6	7½
	7	7½
	7½	6½
	lift	pencil
	8	2
	8½	3
	8	4
*	7½	4
	7	3
	8	2
	lift	pencil
	2½	-2½
	3	-4
	4	-5
	5½	-5
	8	-3½
	-9½	-1½
	lift	pencil
	3½	-2
	2½	-2½
	2	-3½
	lift	pencil
	9	-1
	9½	-1½
	9½	-2

*SHADE IN
THESE AREAS

SECTION C

This section contains an answer sheet for all of the direction sheets given in this booklet. These can be thermofaxed on transparency overlays for ease in checking the students' work.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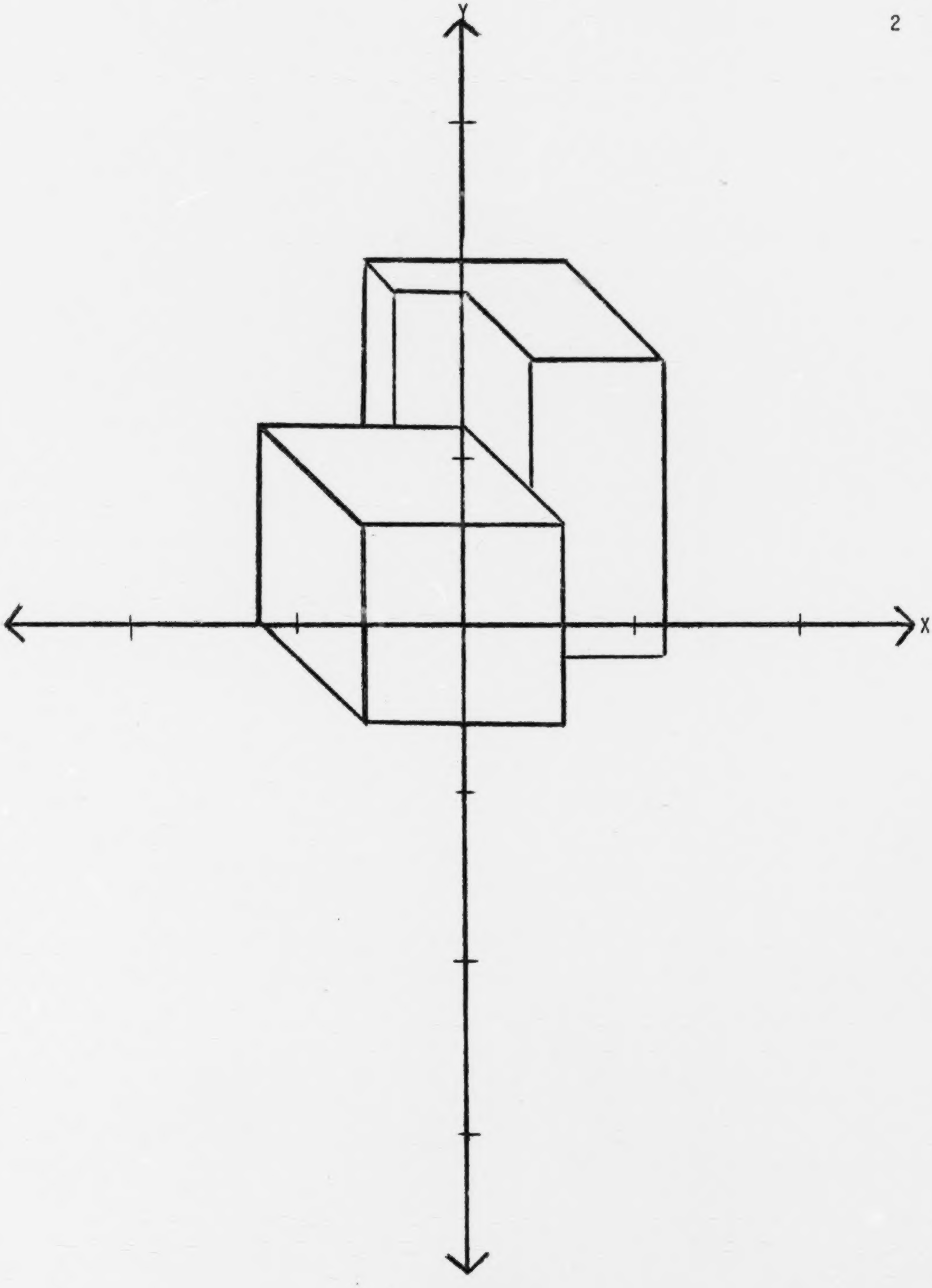
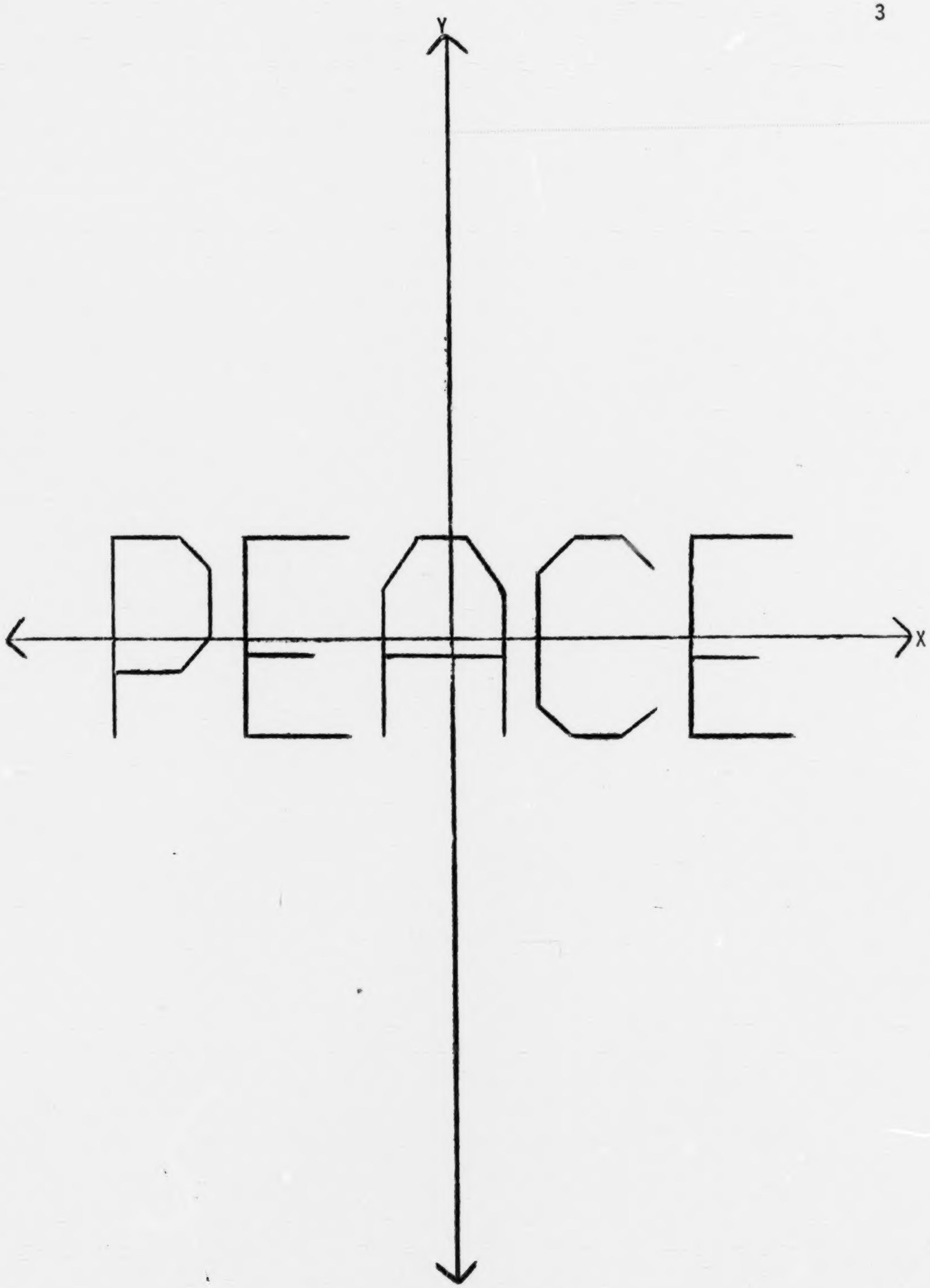
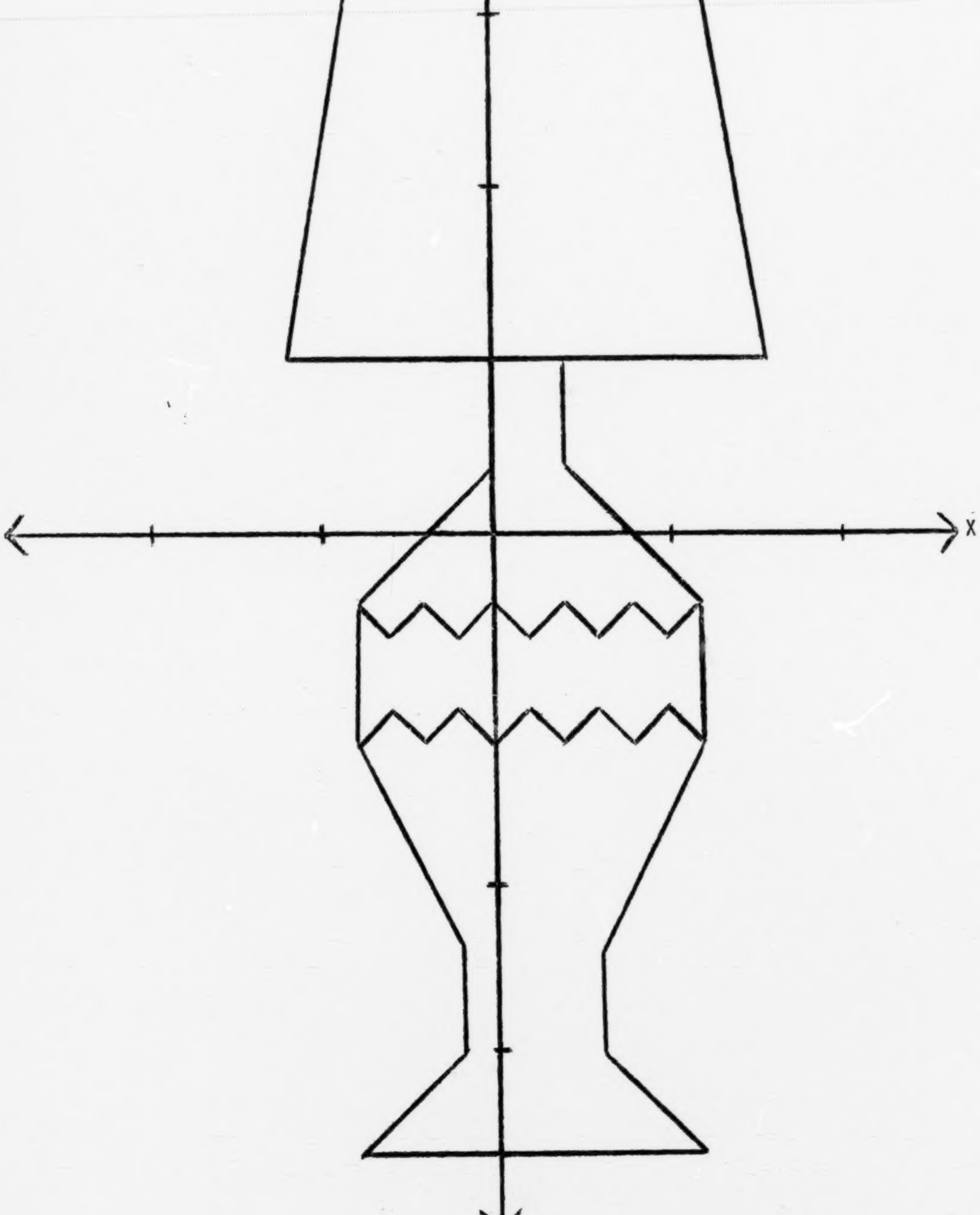
Y
X

2




Y
X
PEACE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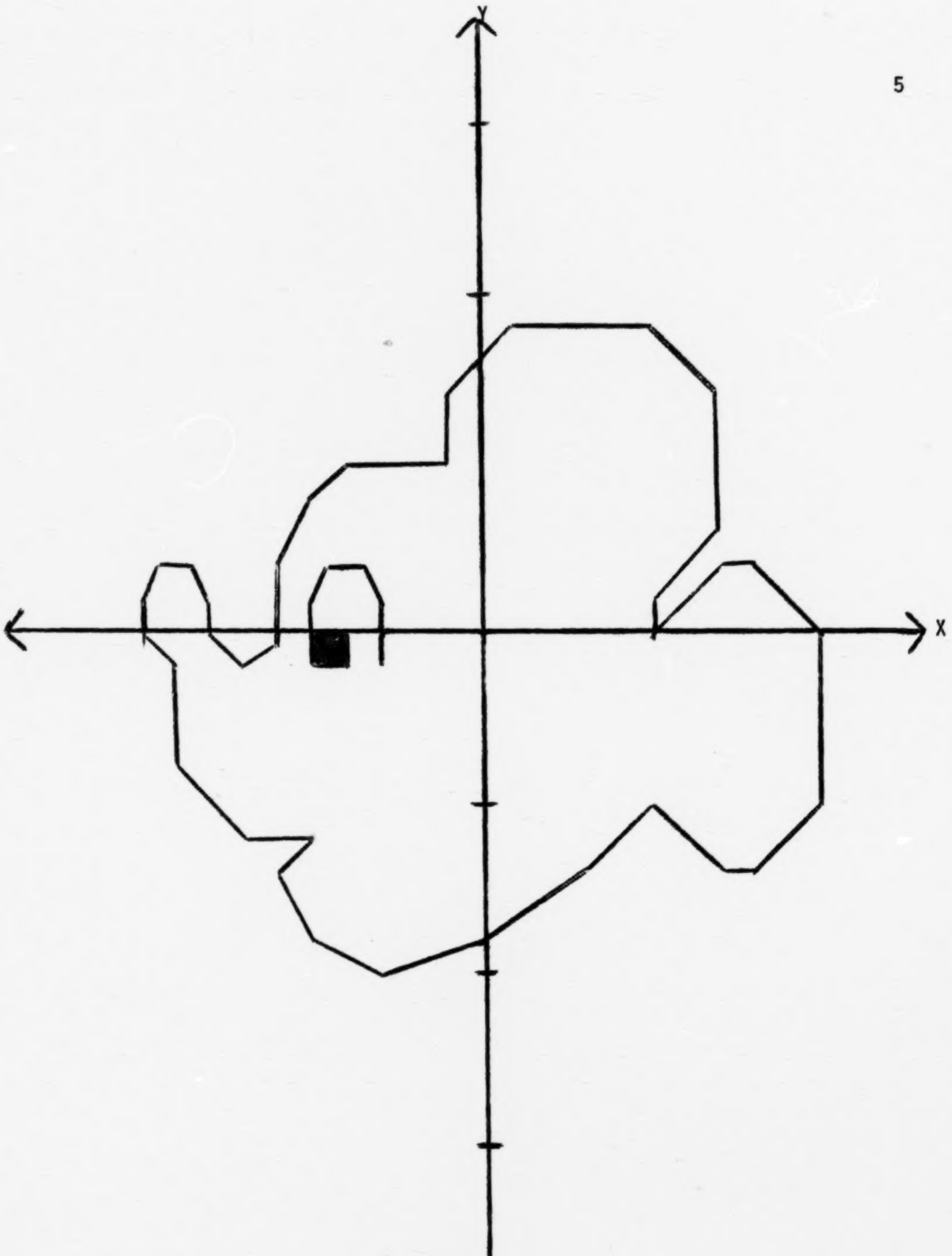
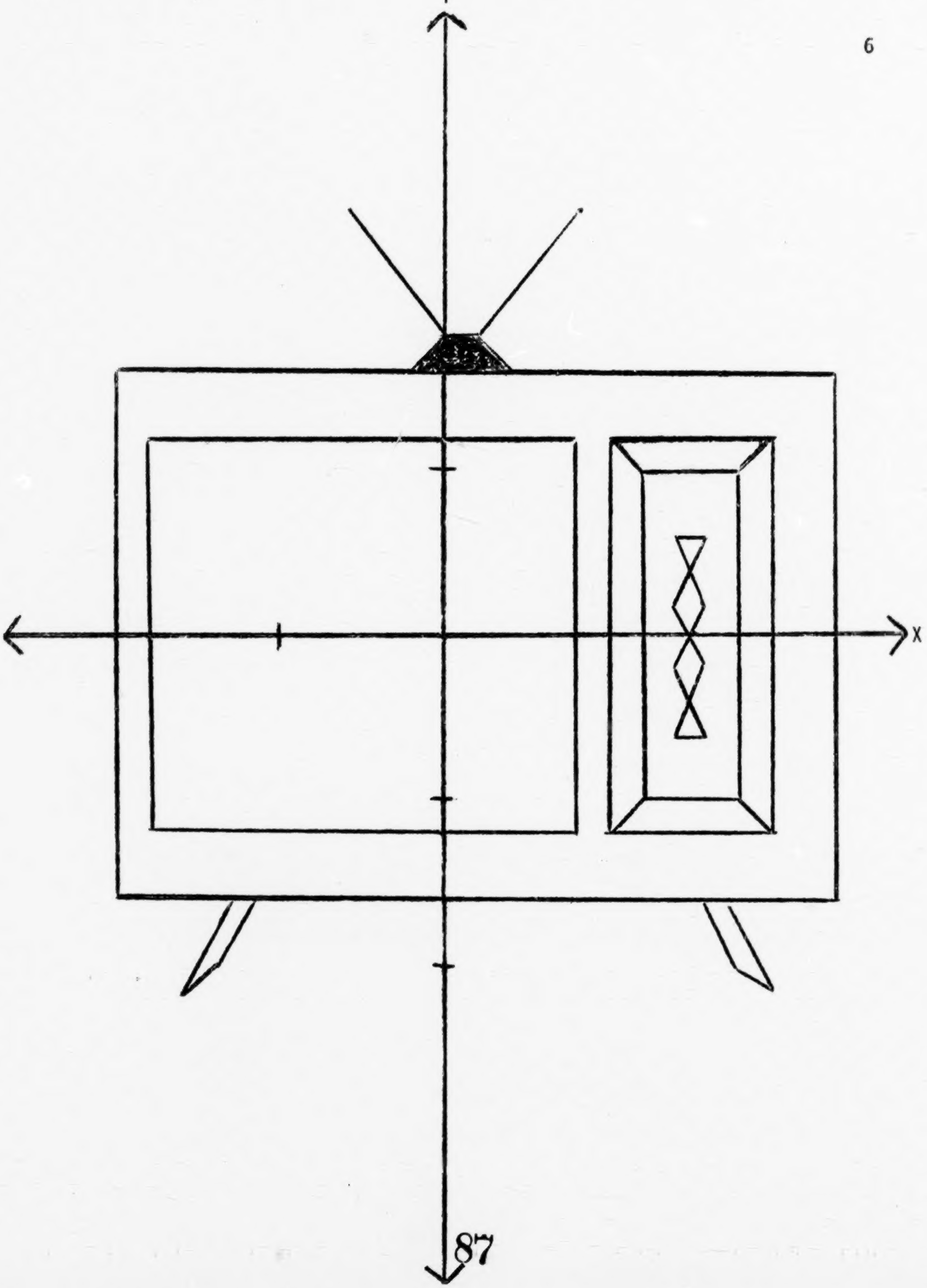
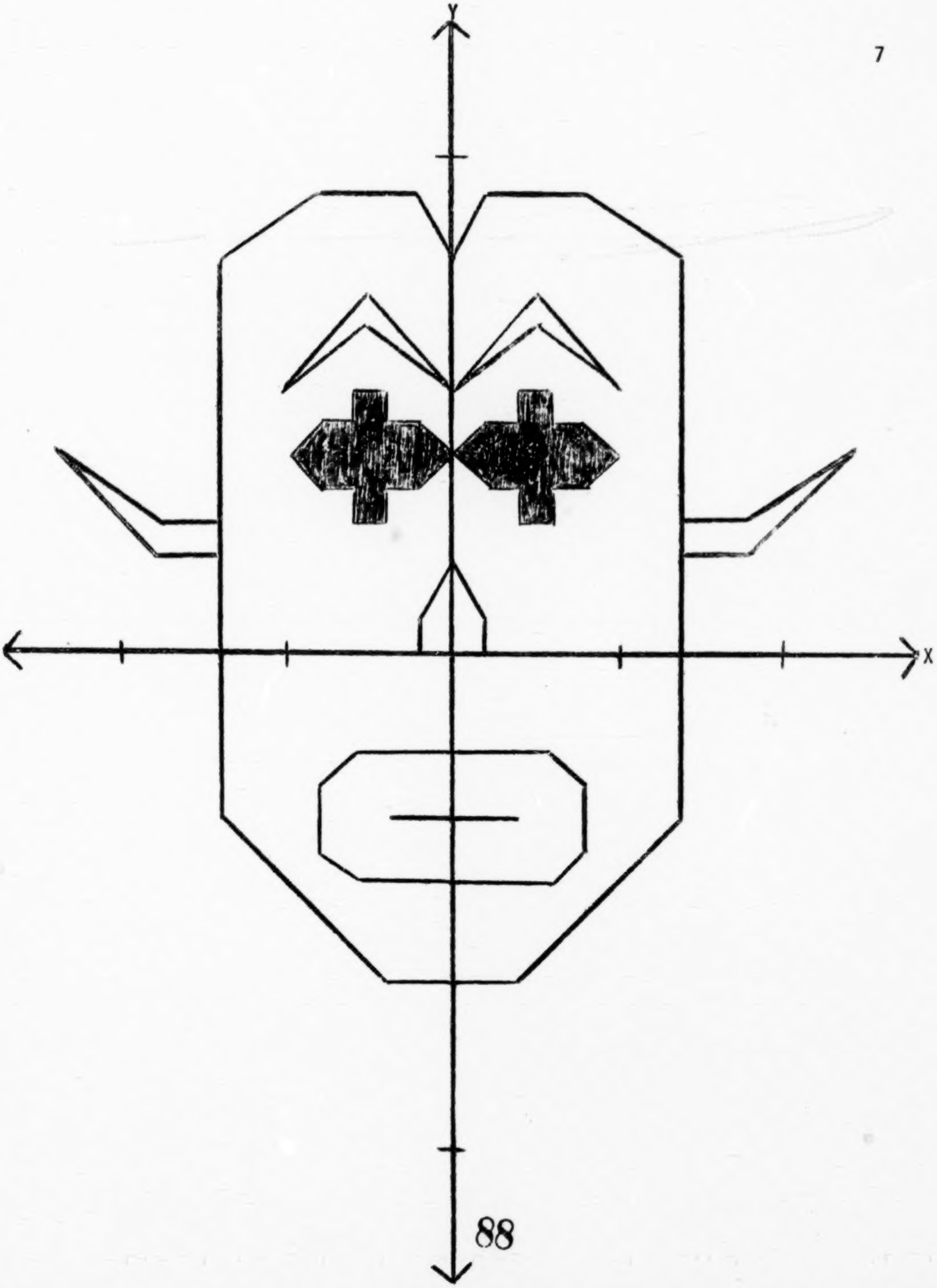
X

5




Y
X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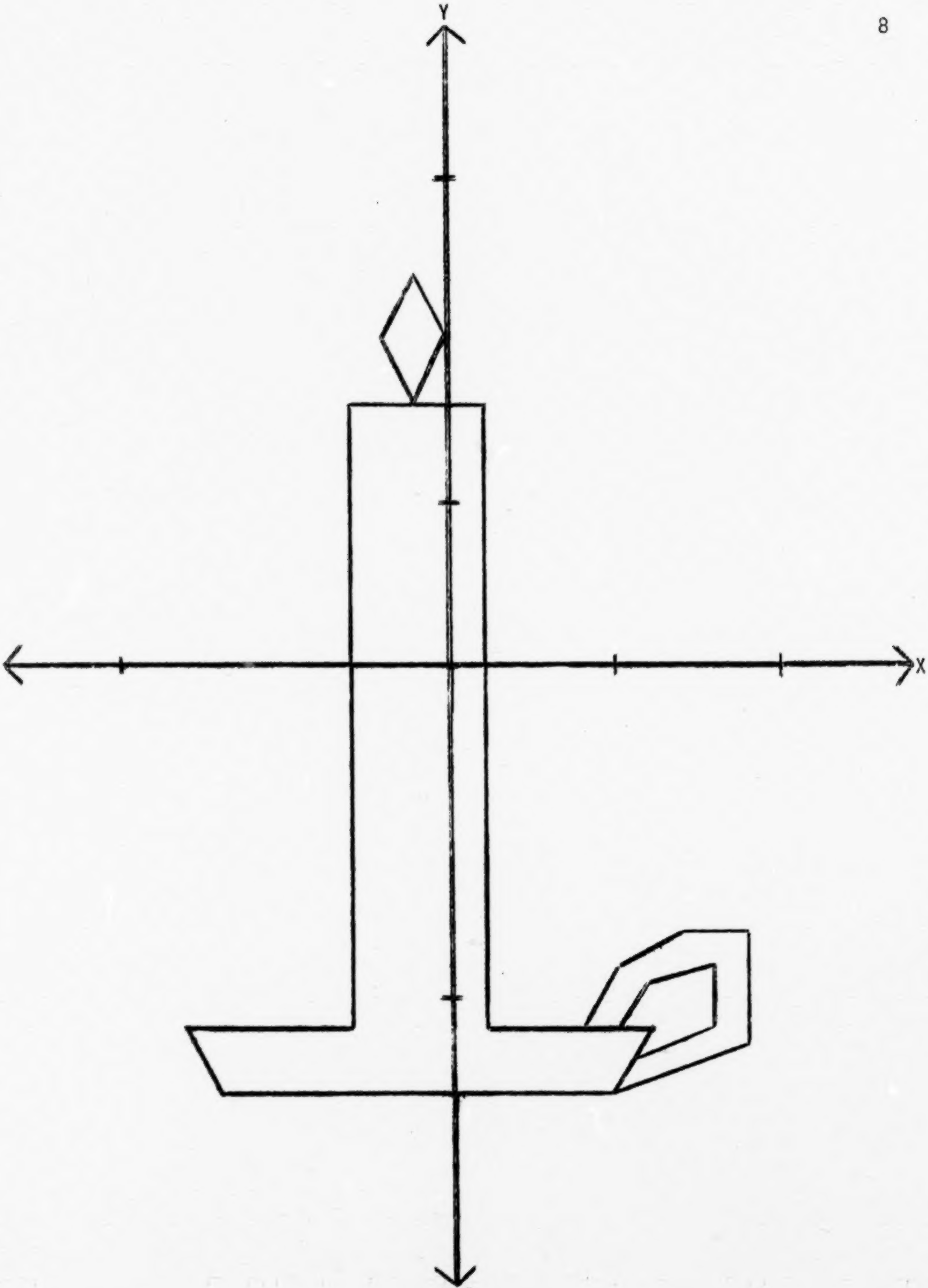
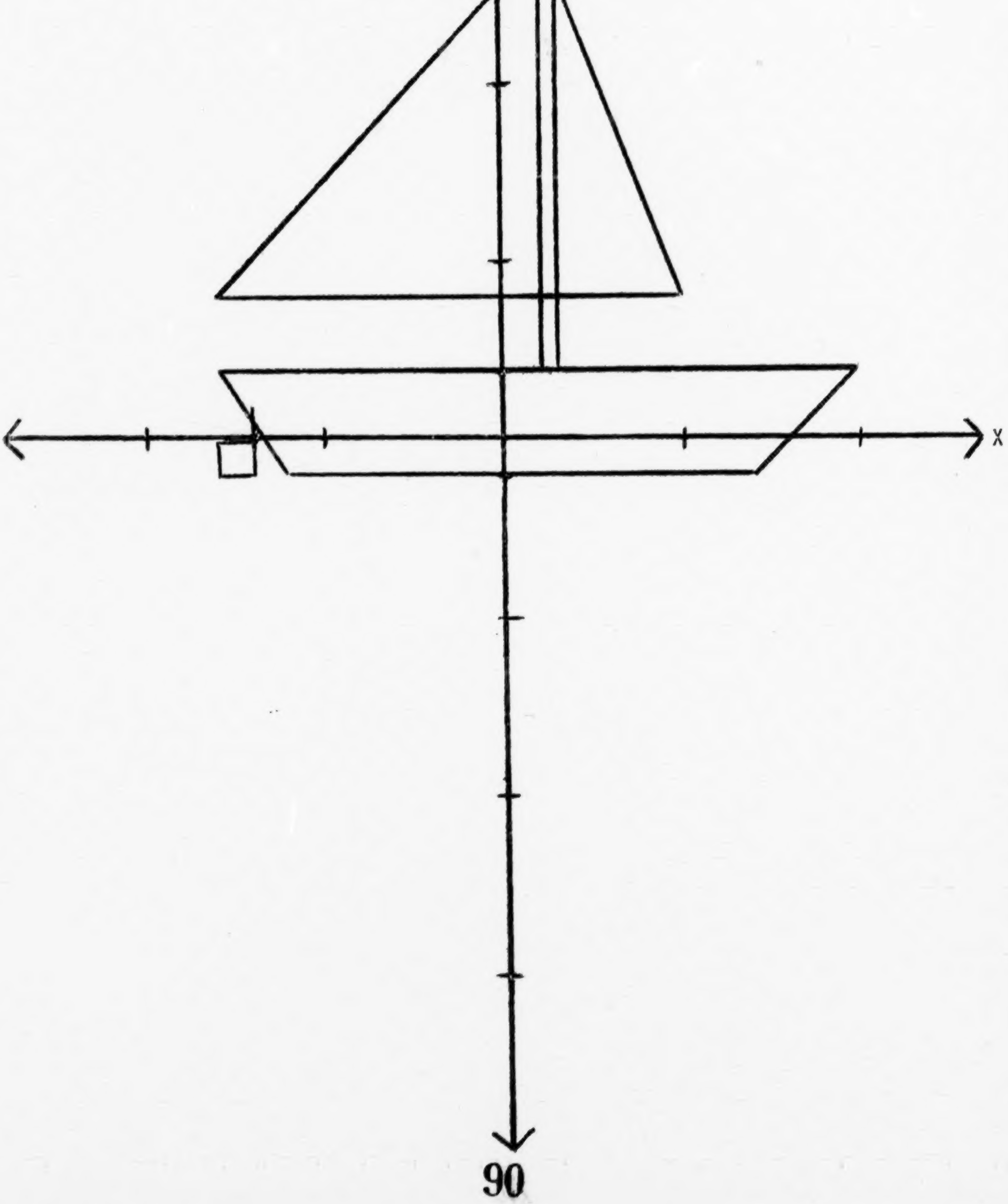
x

8

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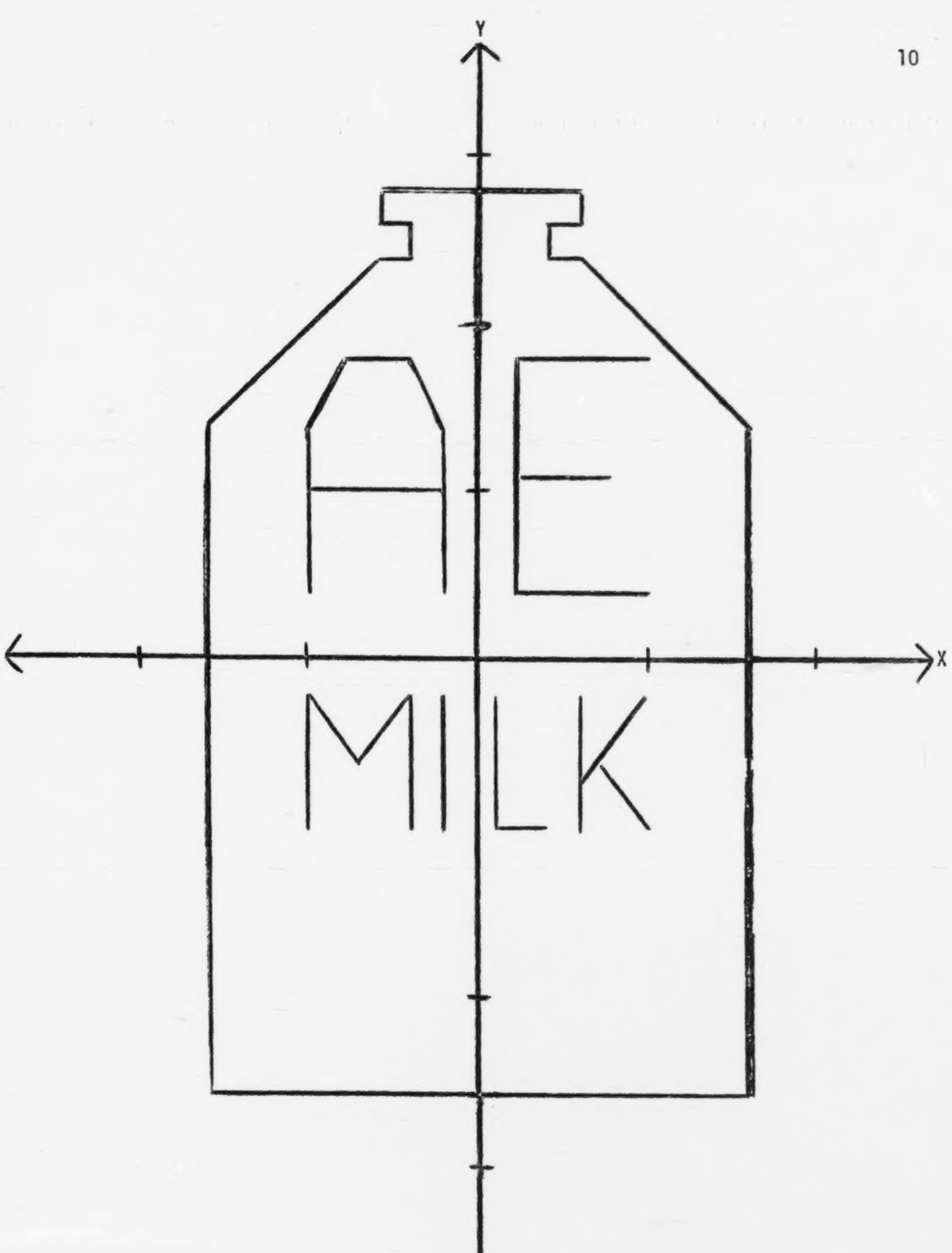
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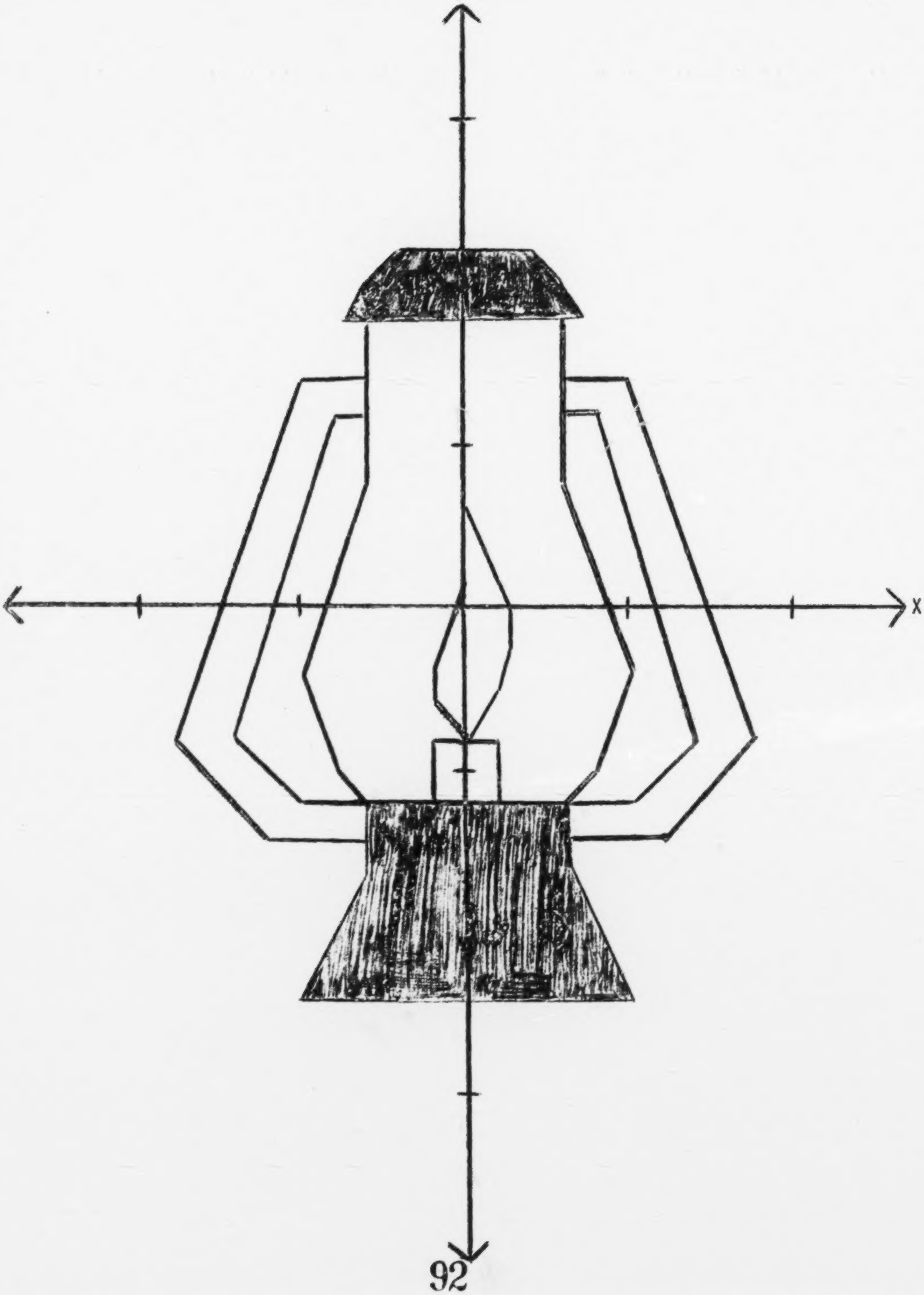
X
AE
MILK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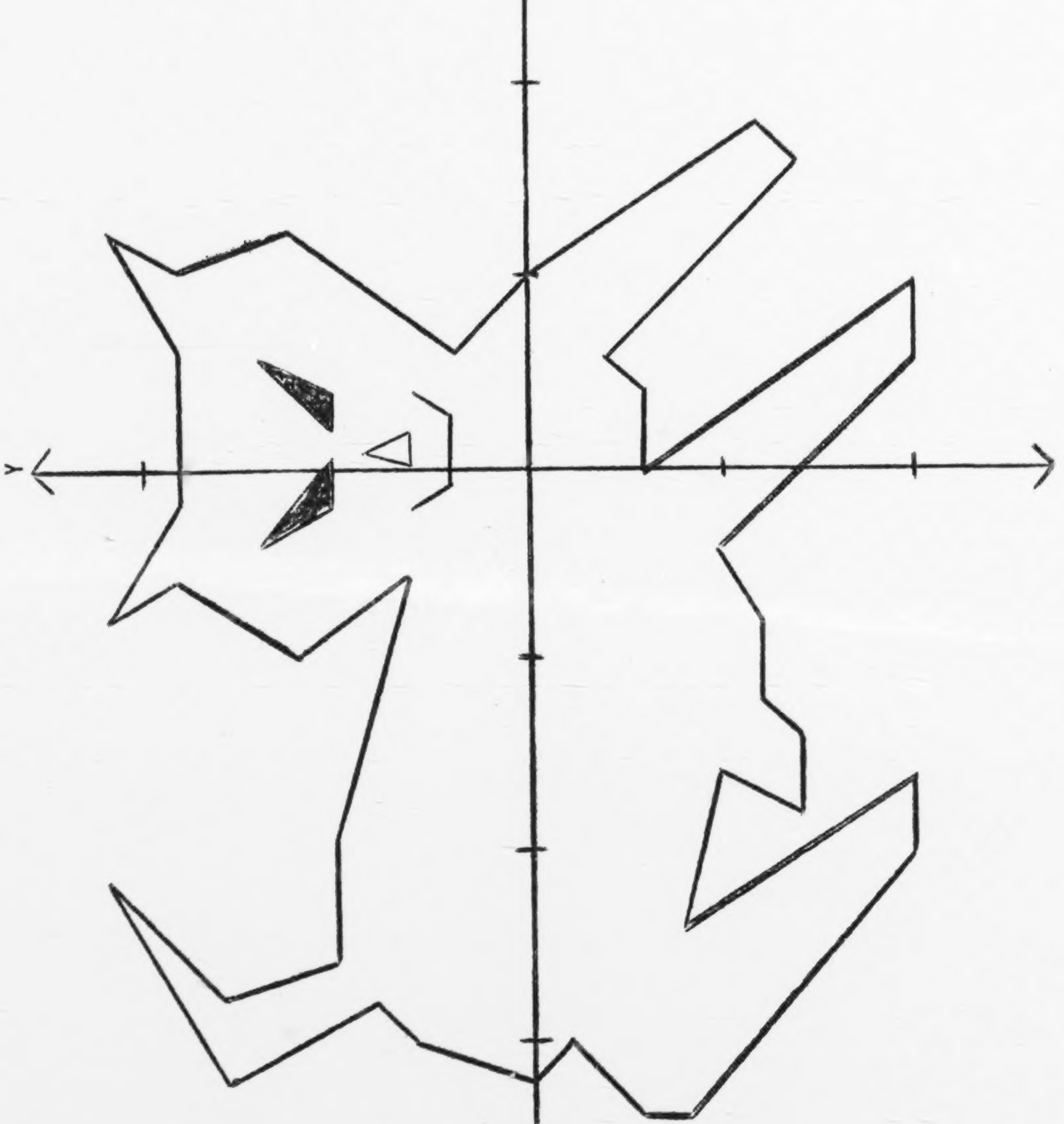
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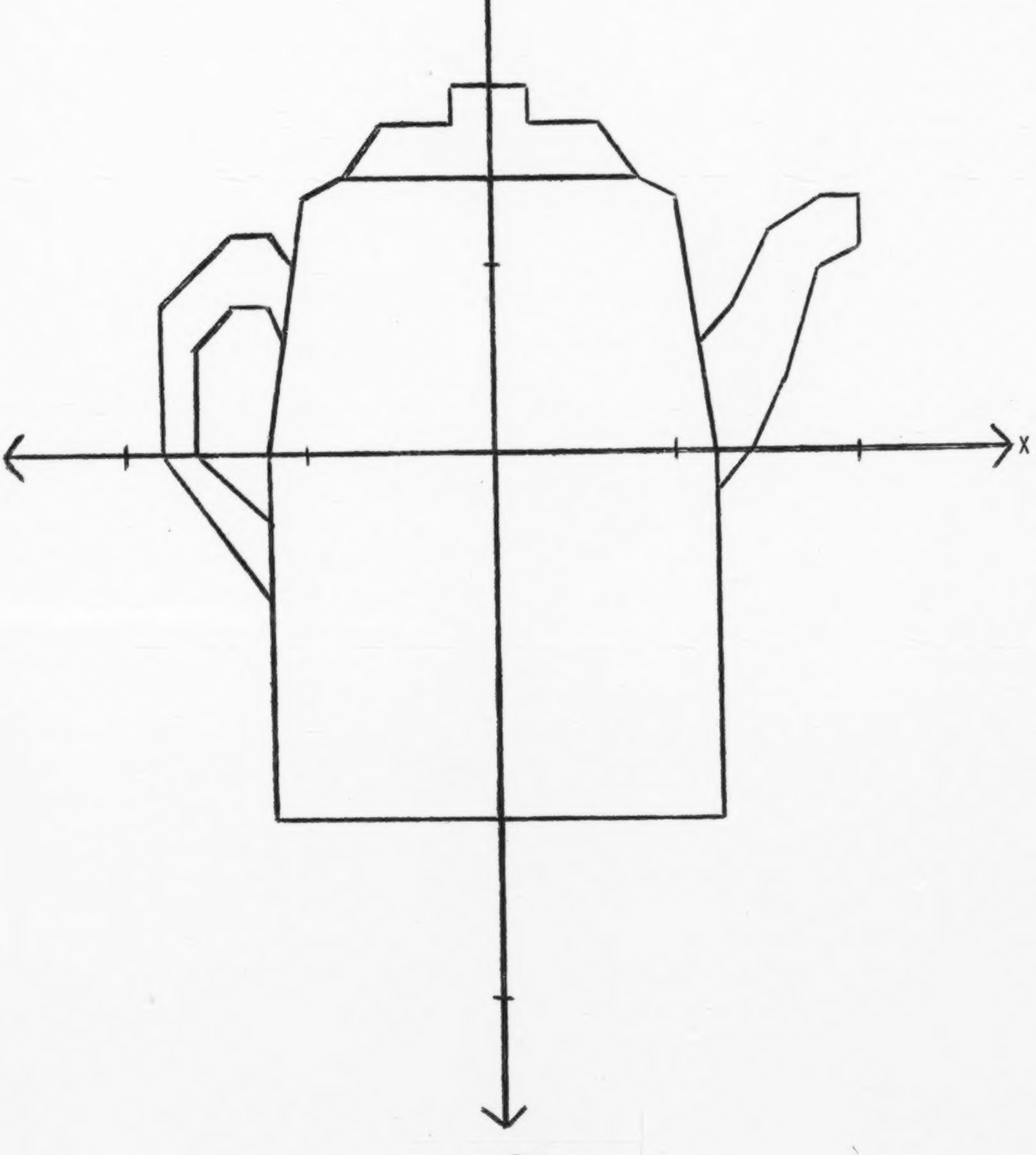
y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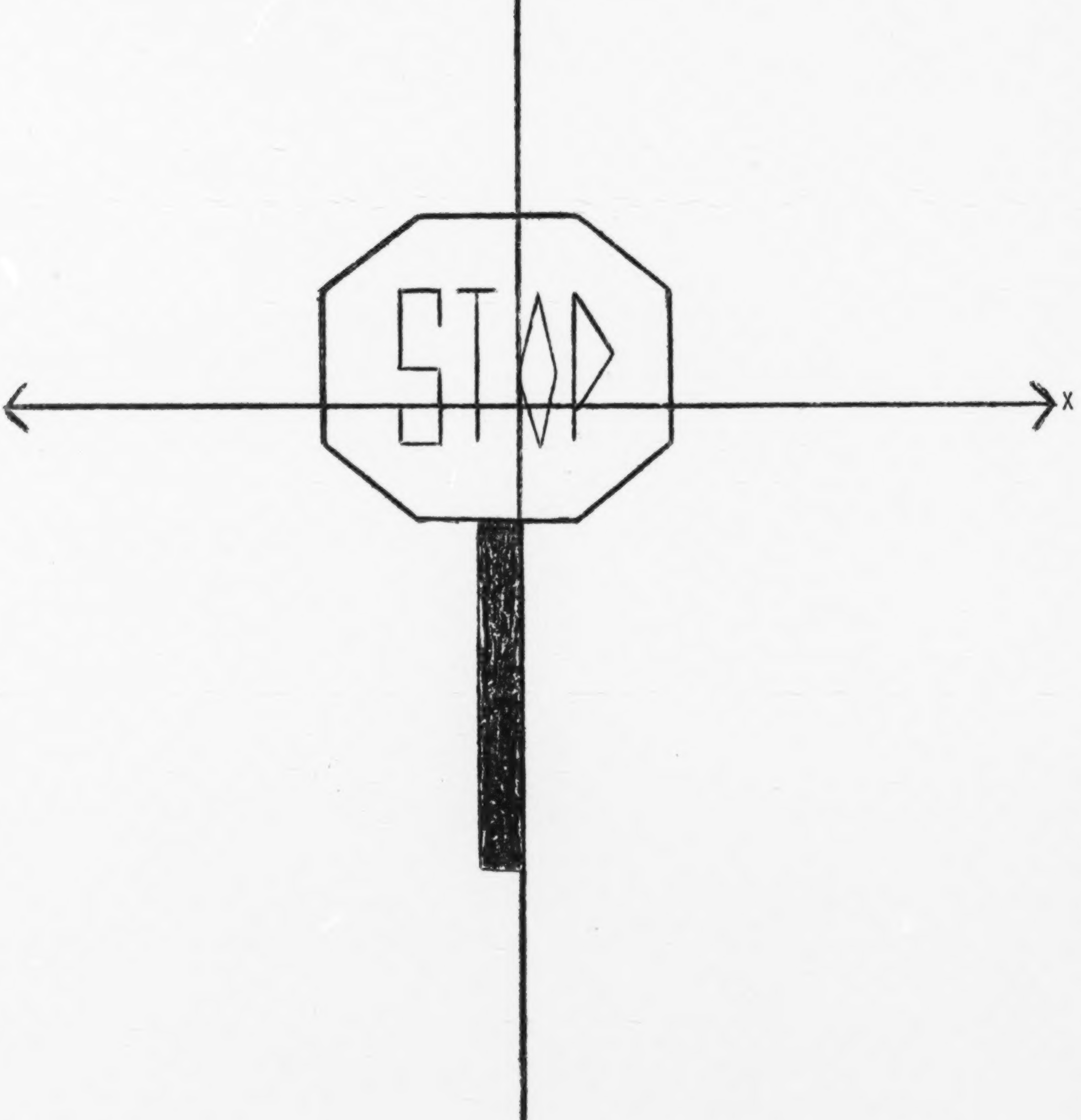
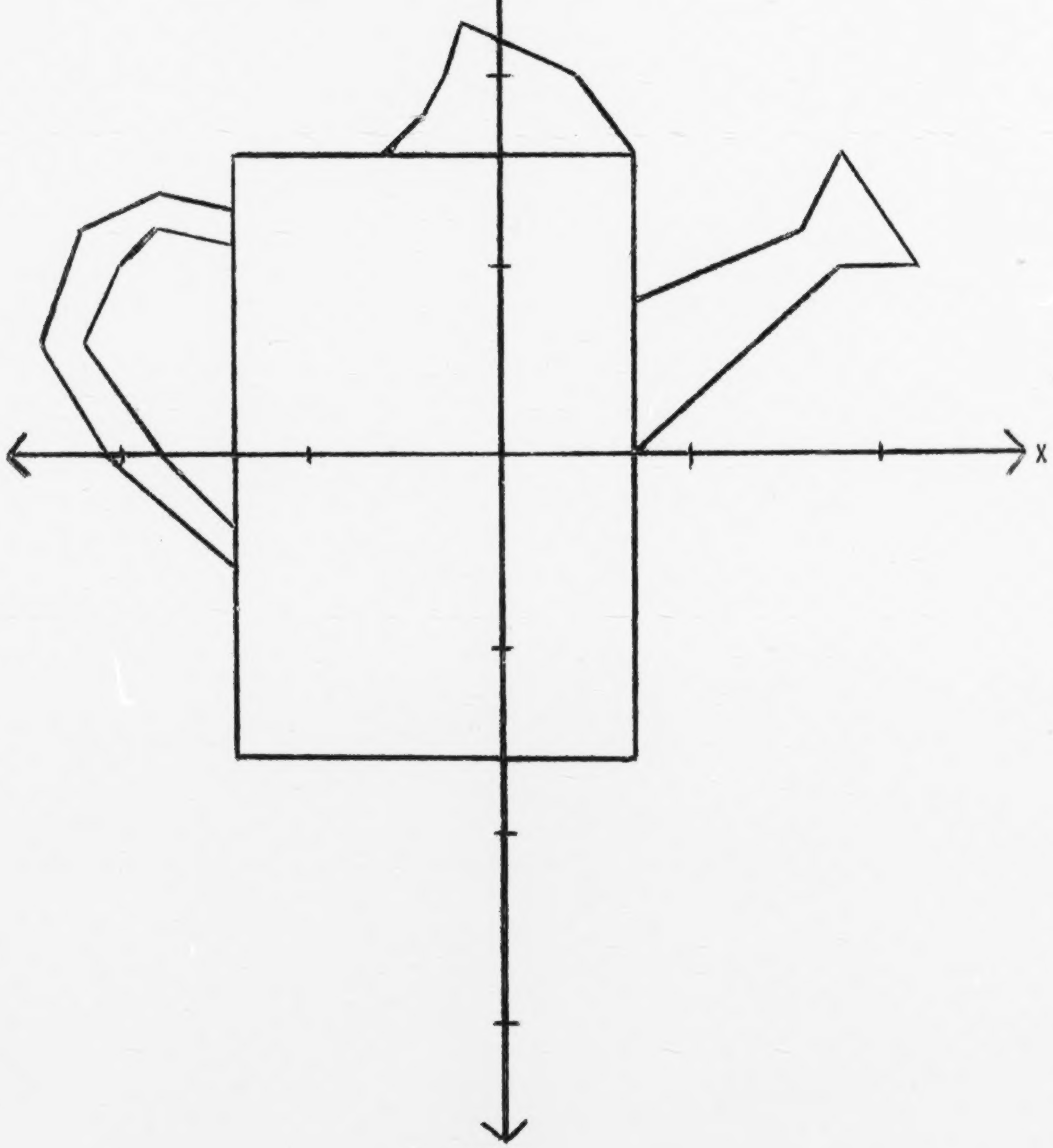
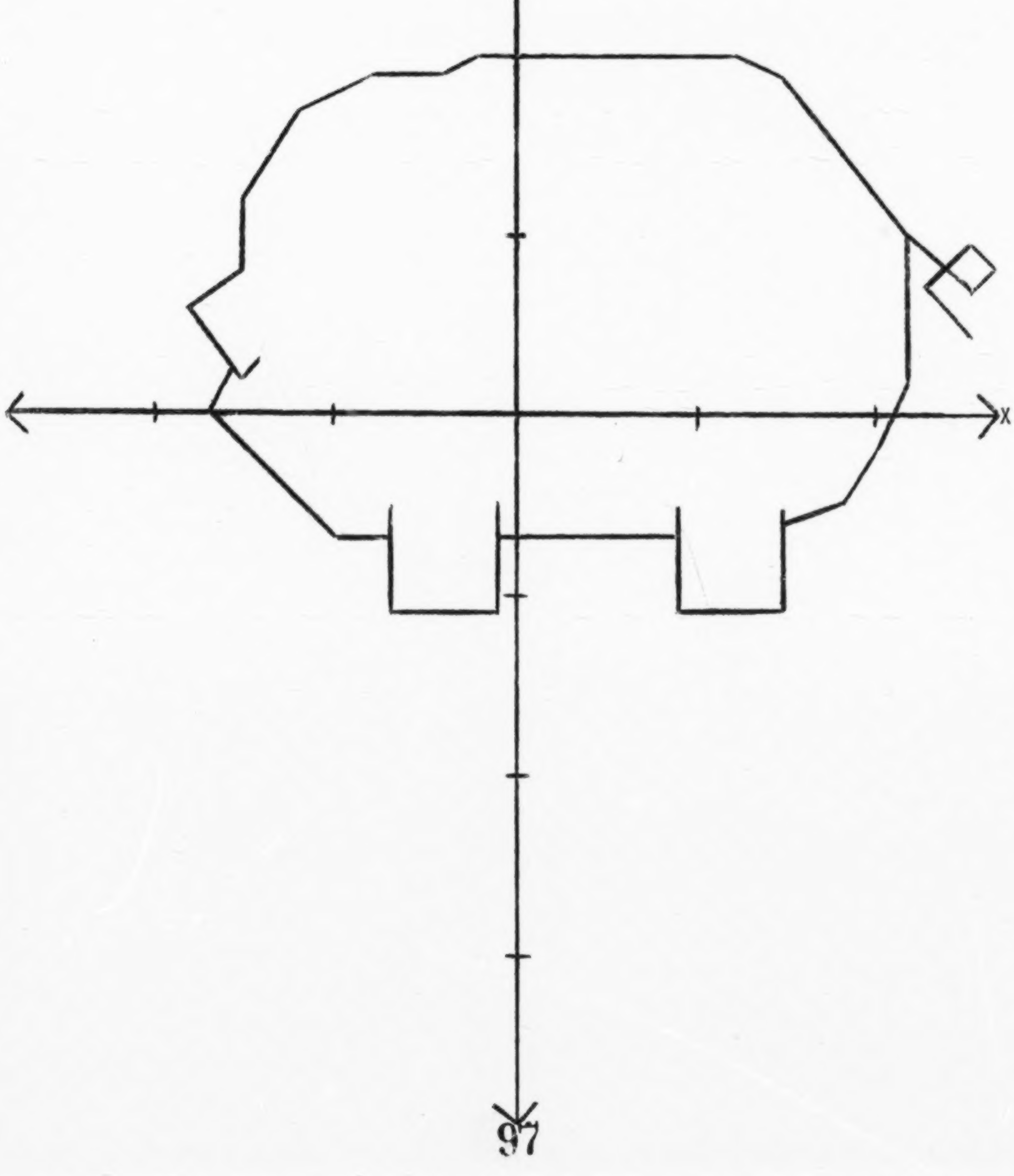
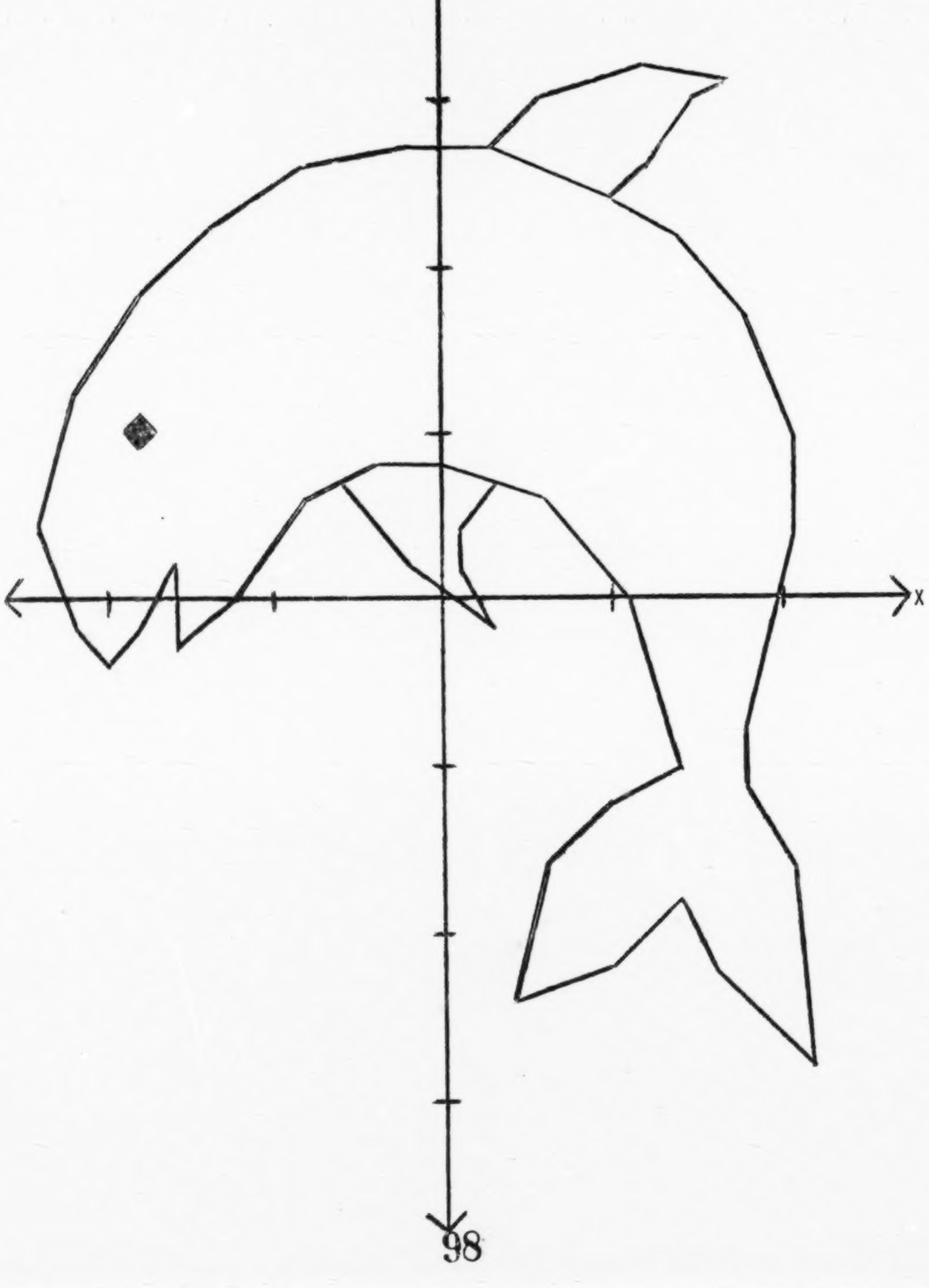
X


Y
X
STOP


Y
X


Y
X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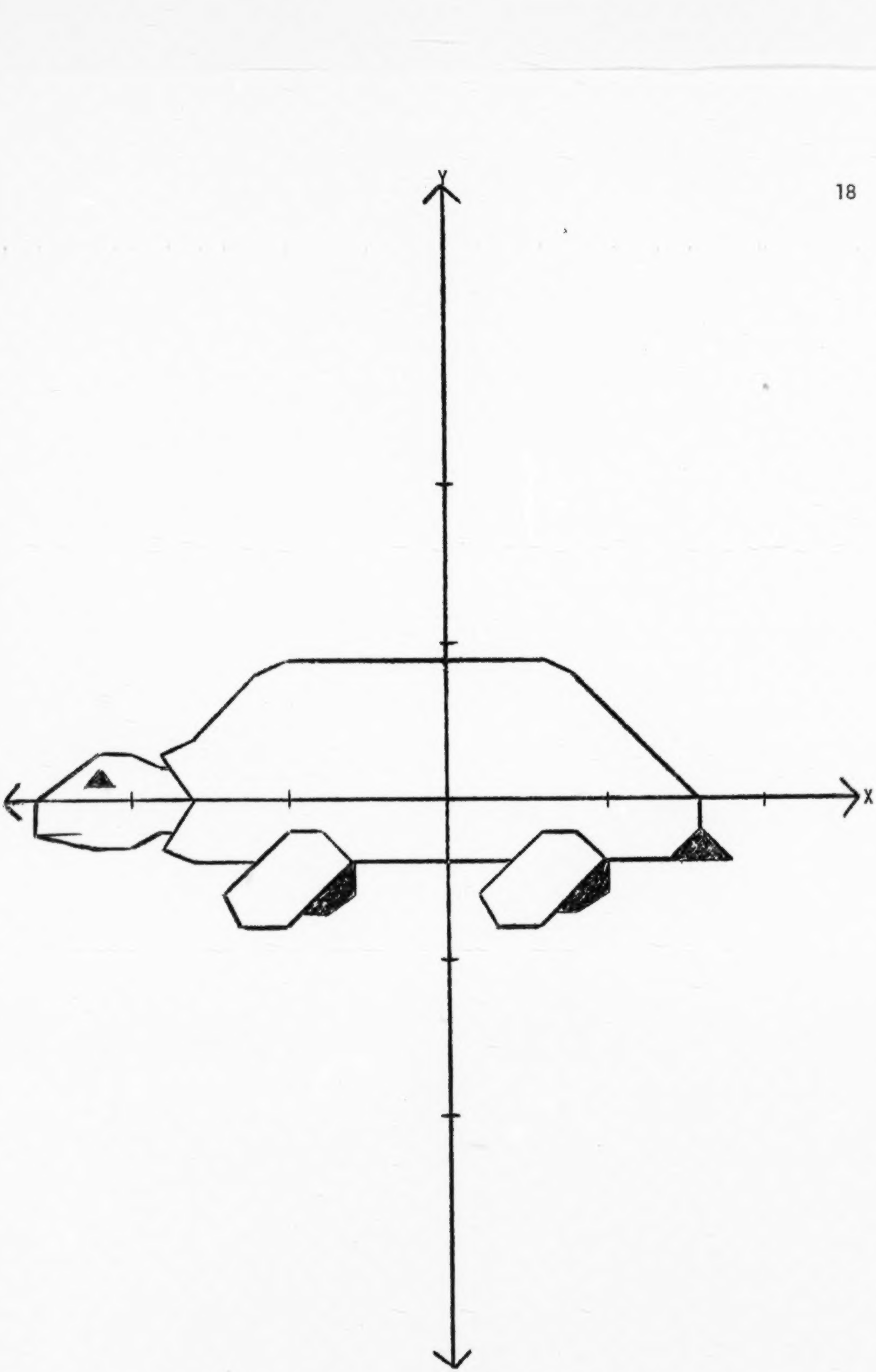
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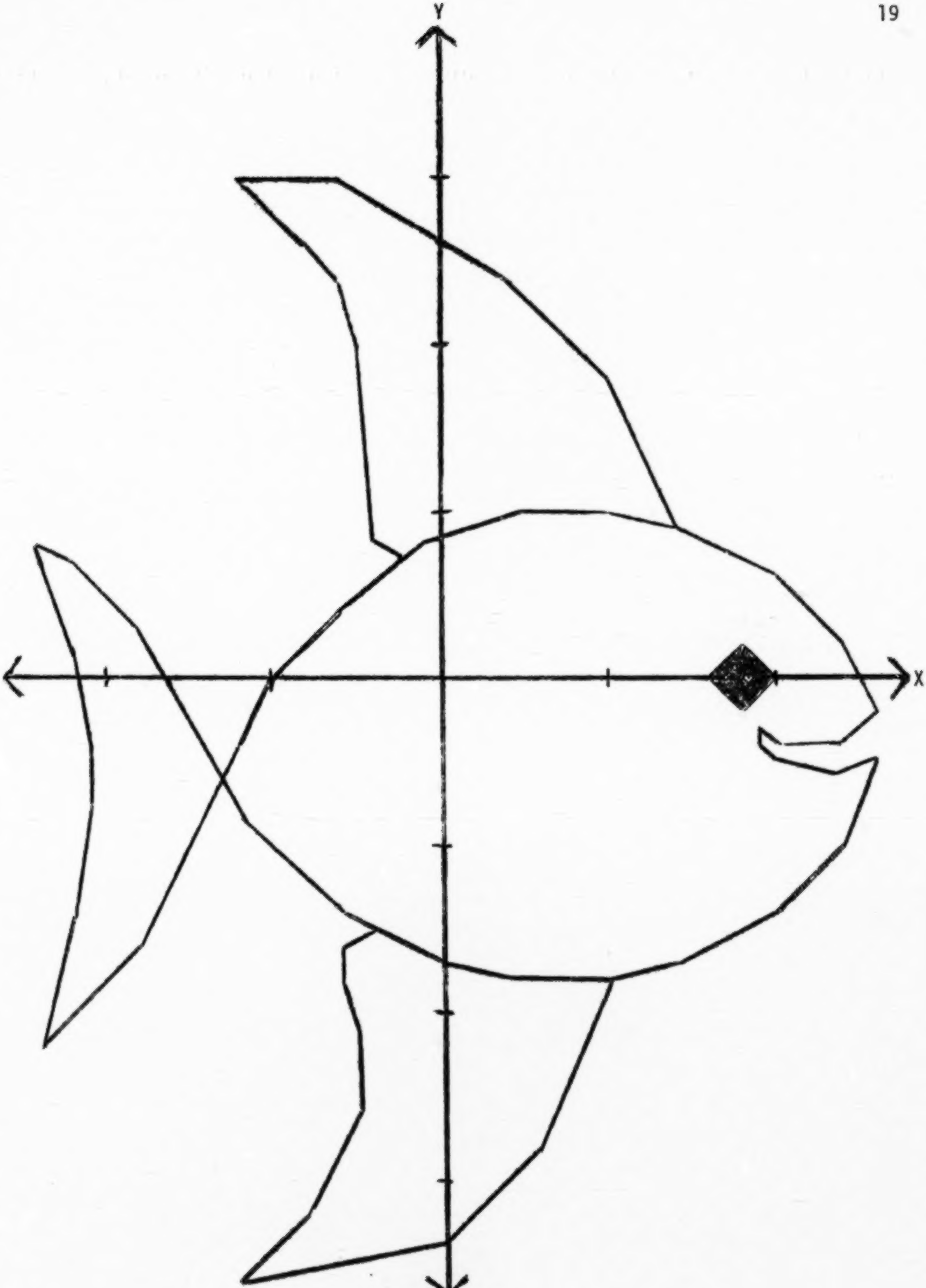
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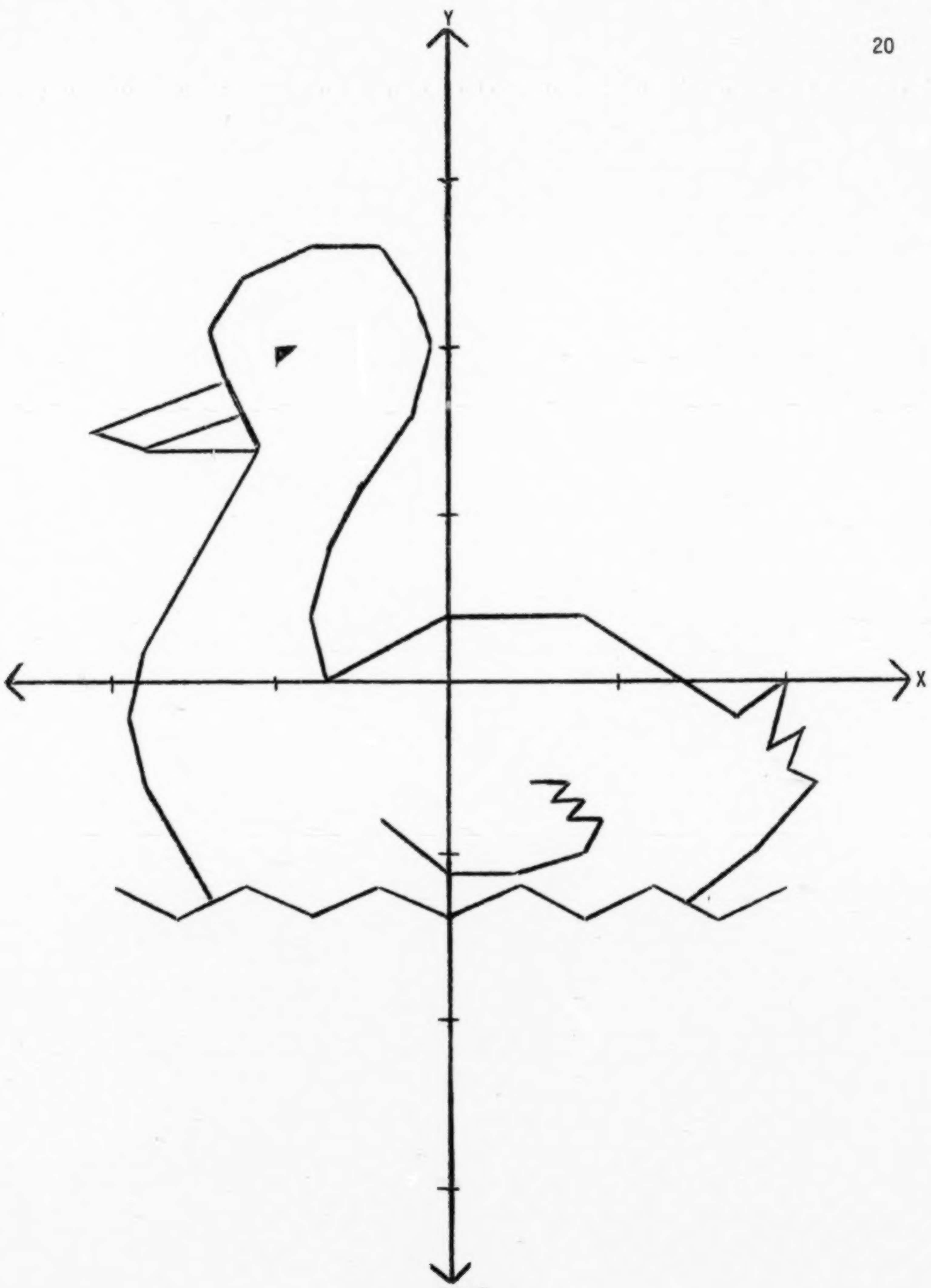
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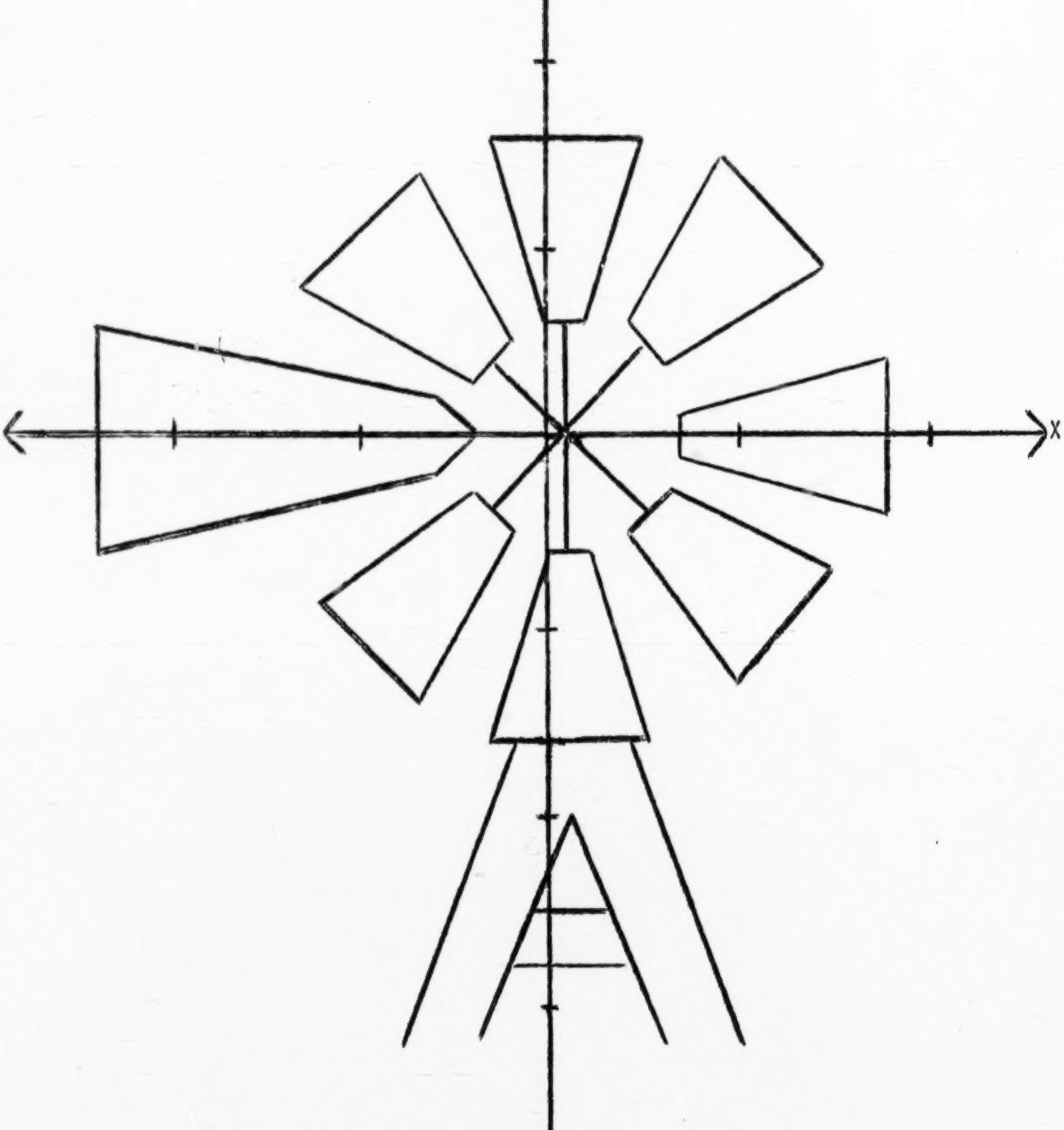
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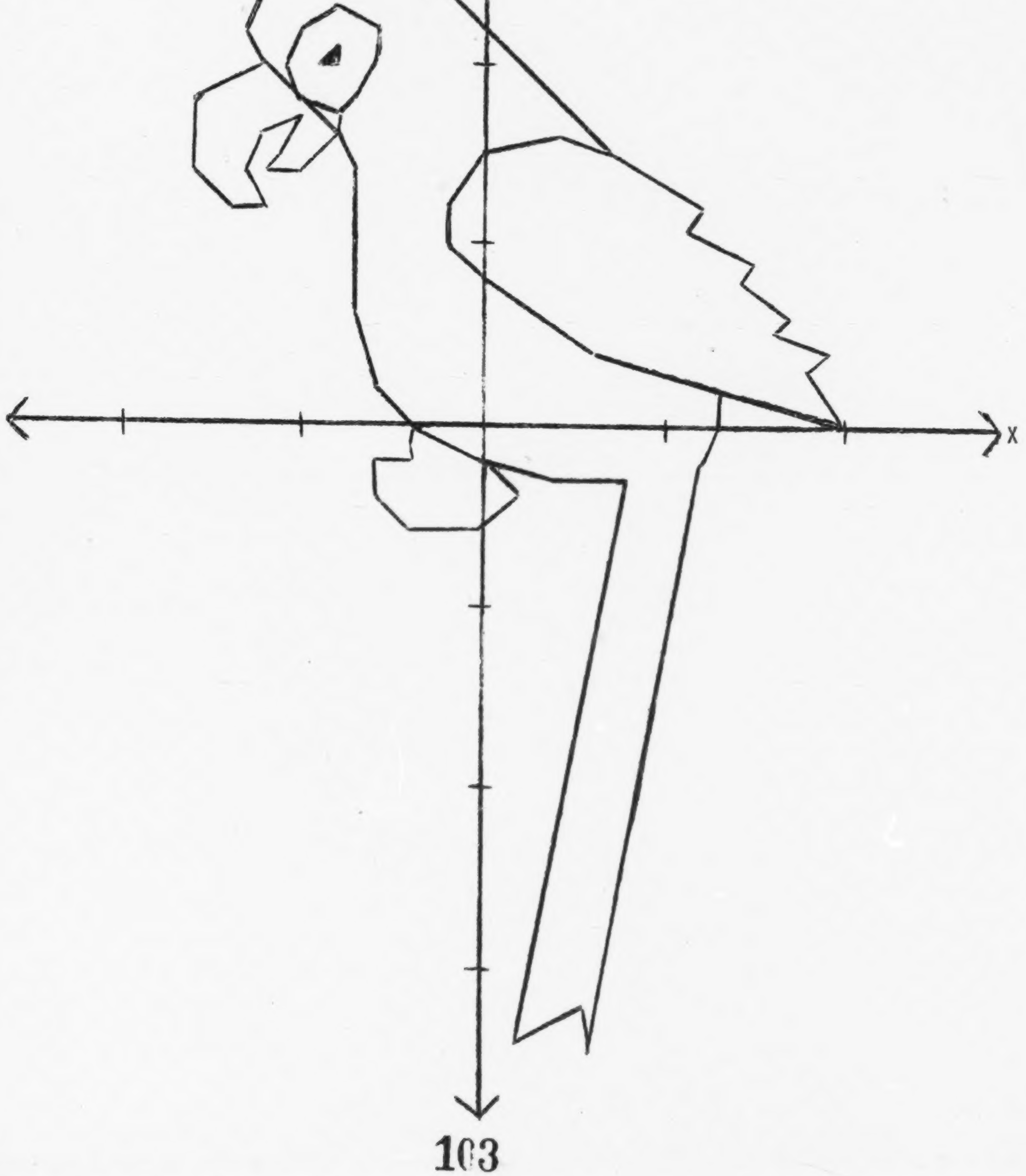
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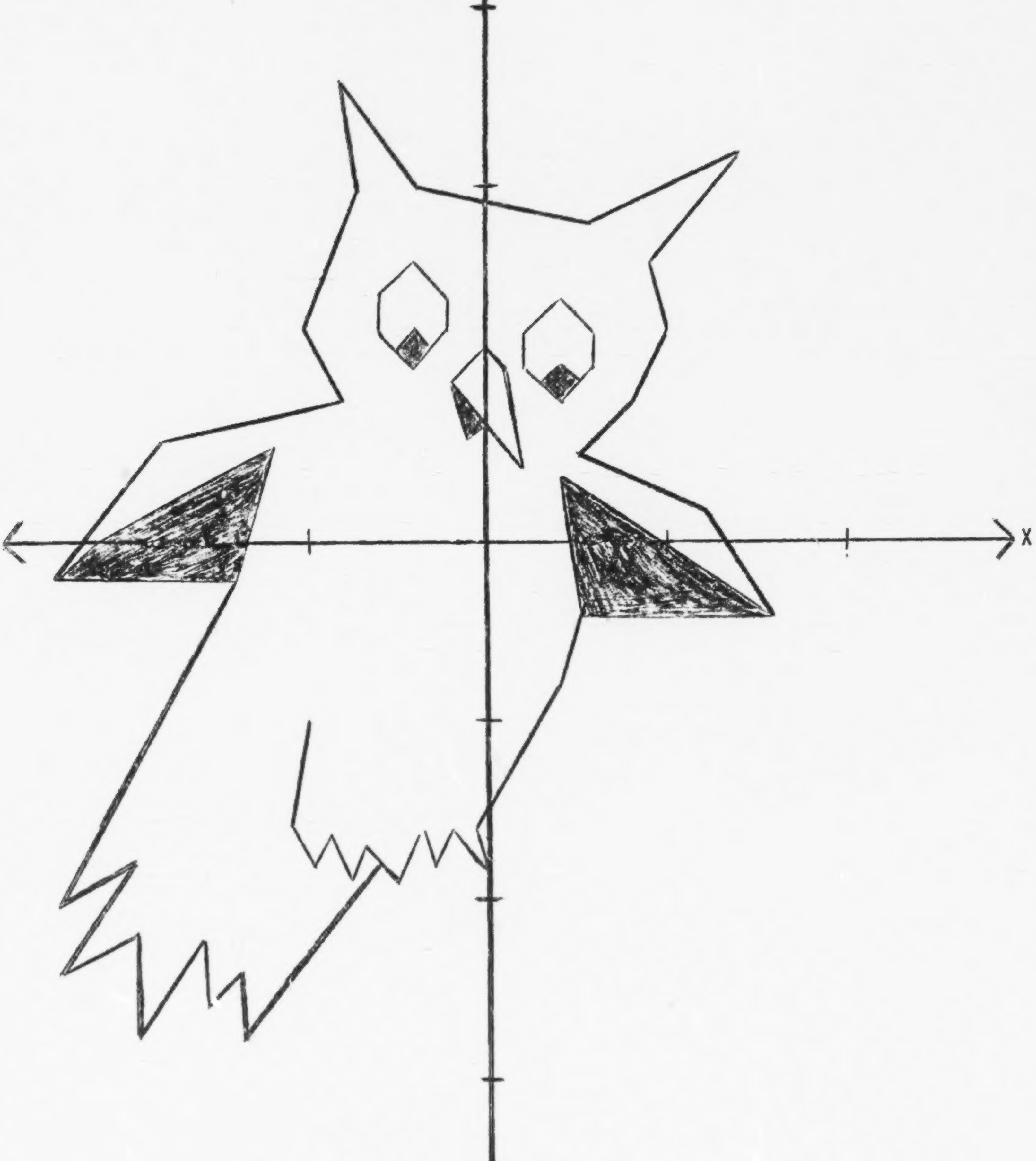
X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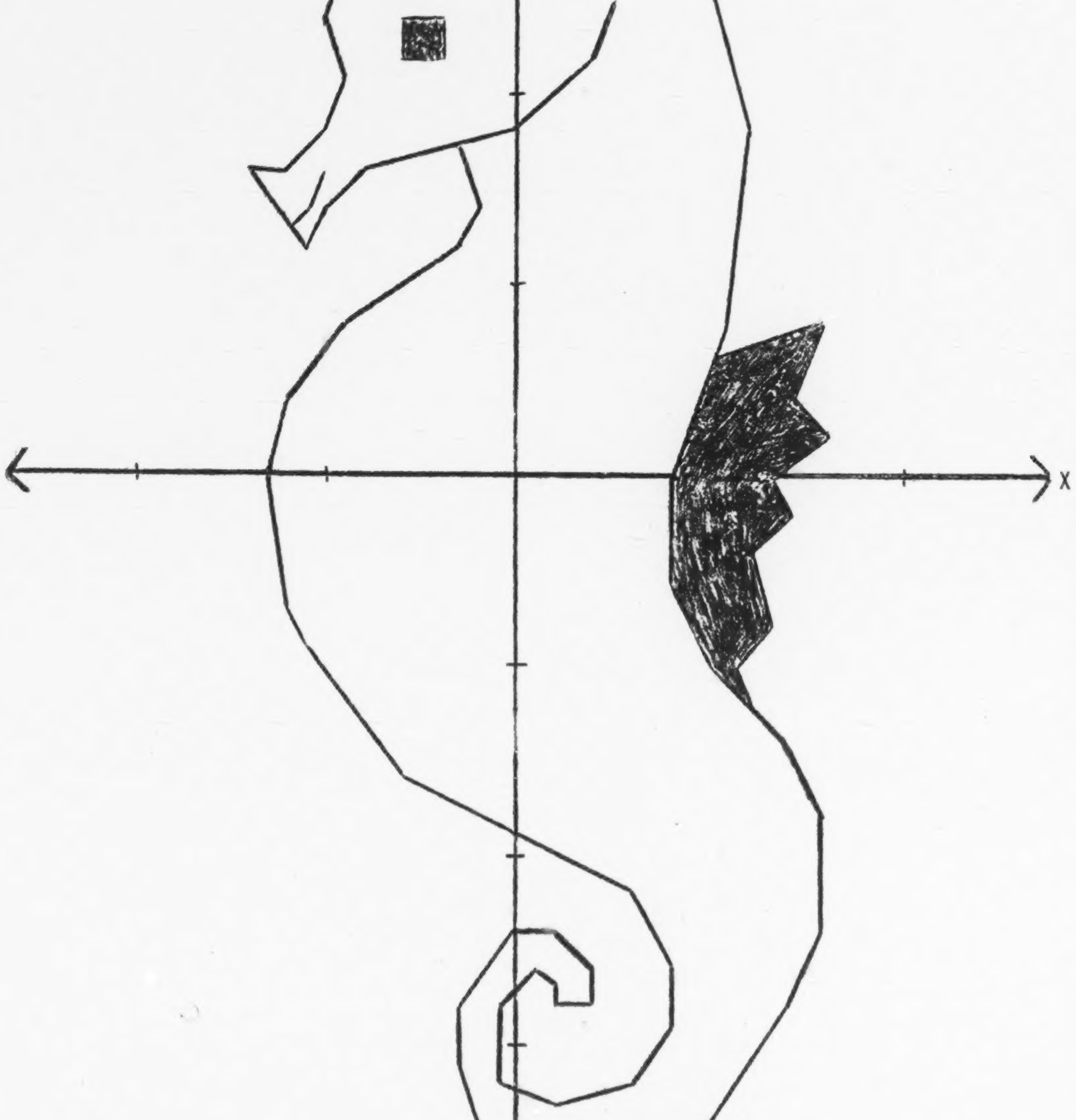
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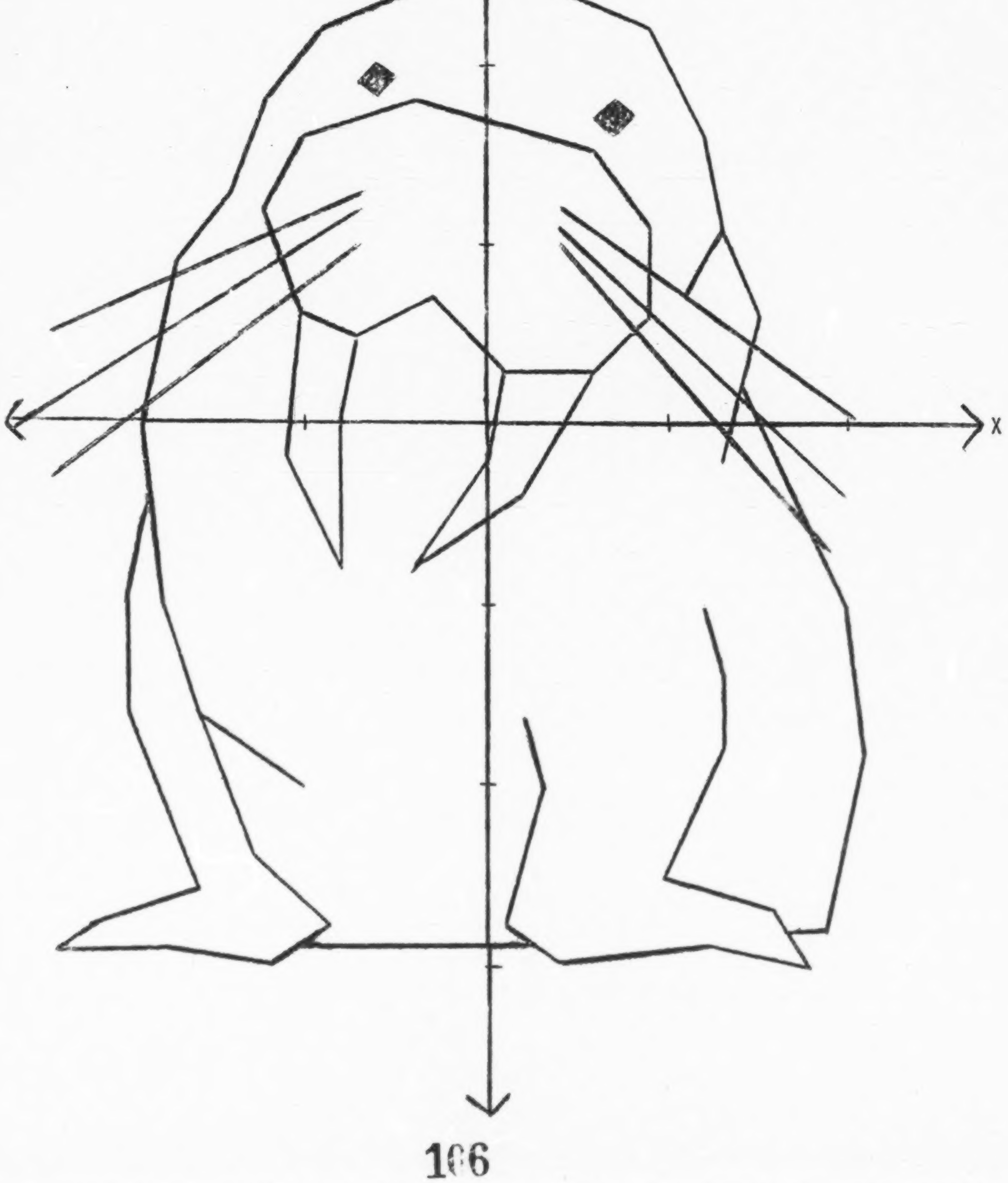
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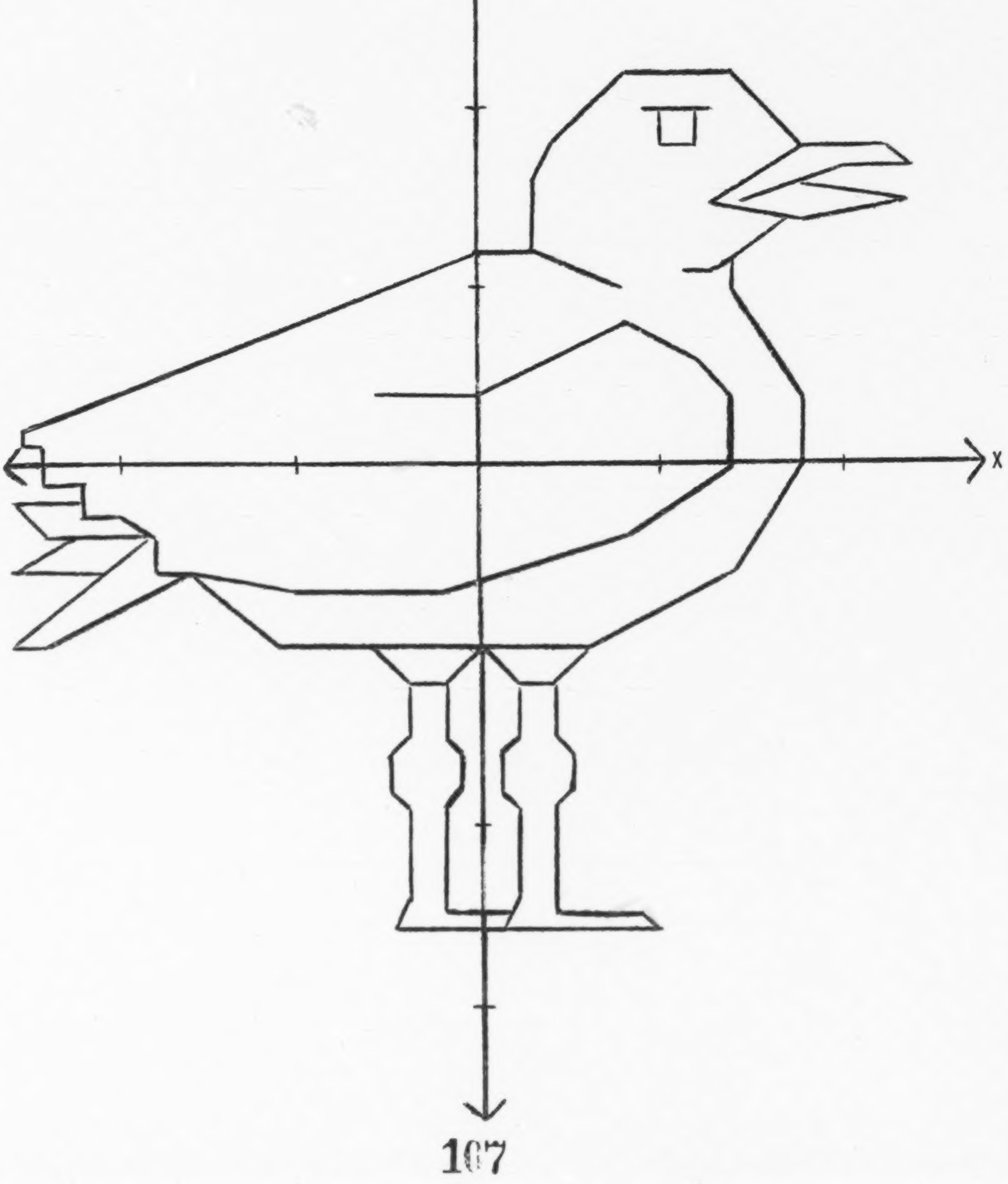
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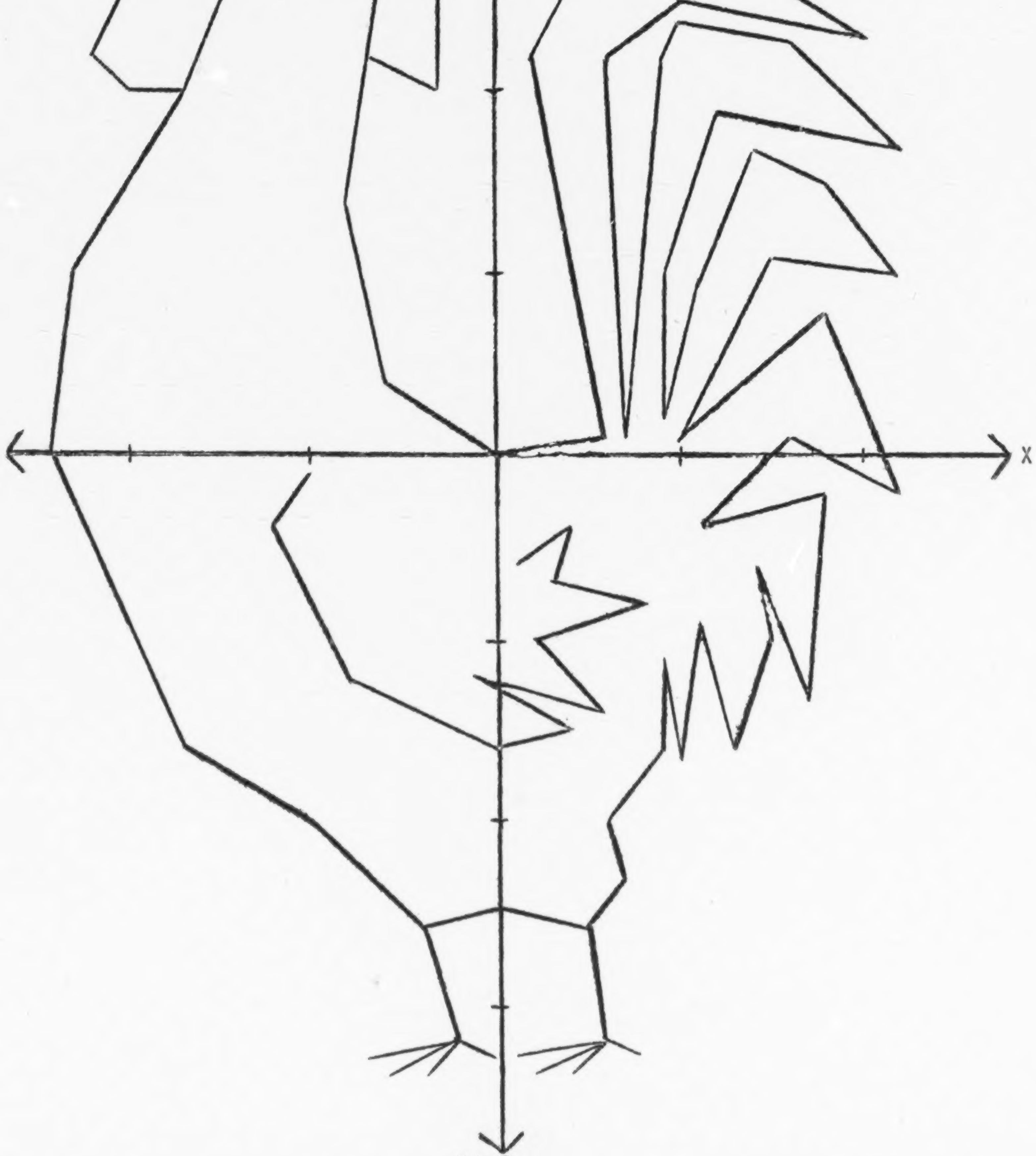
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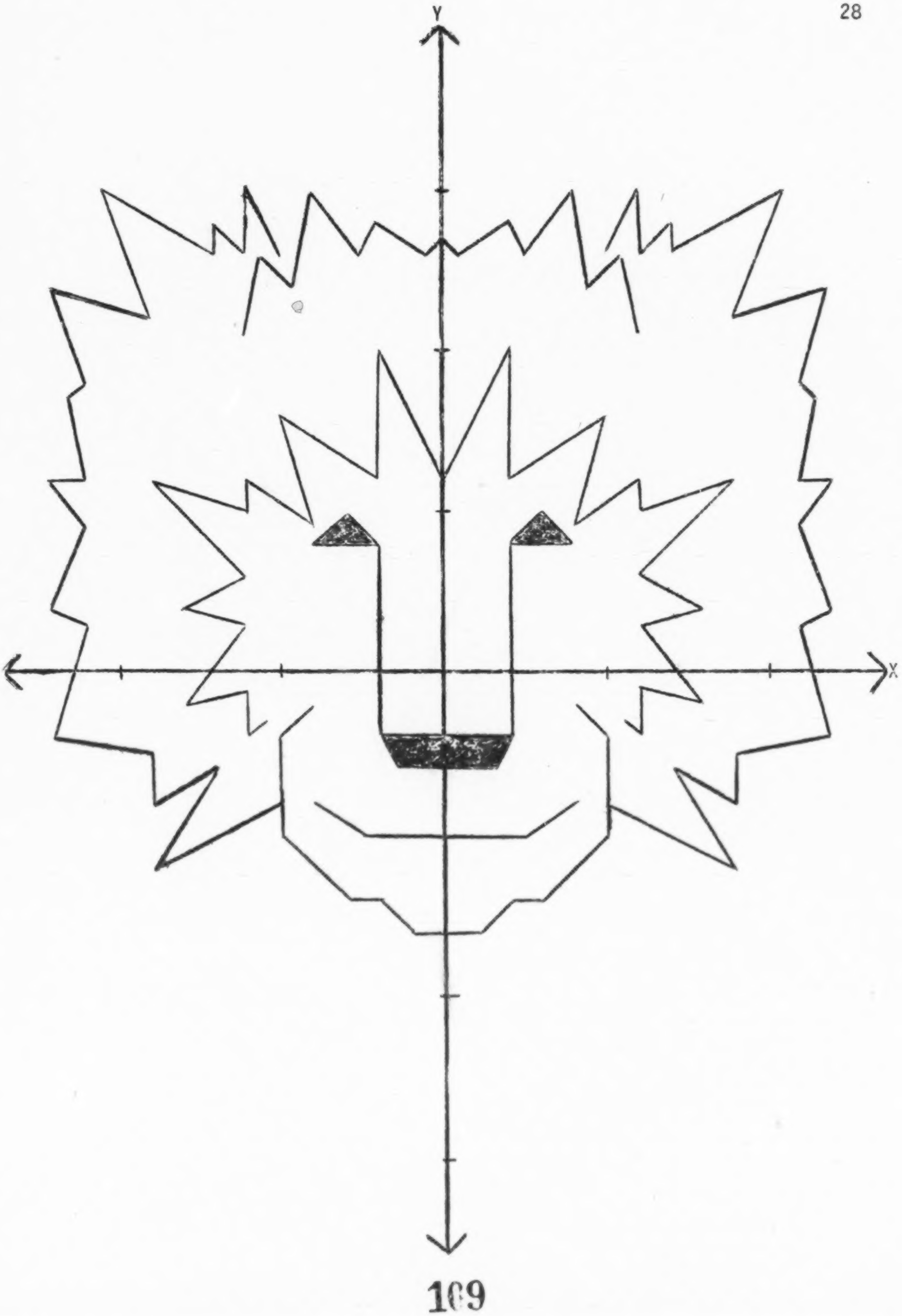
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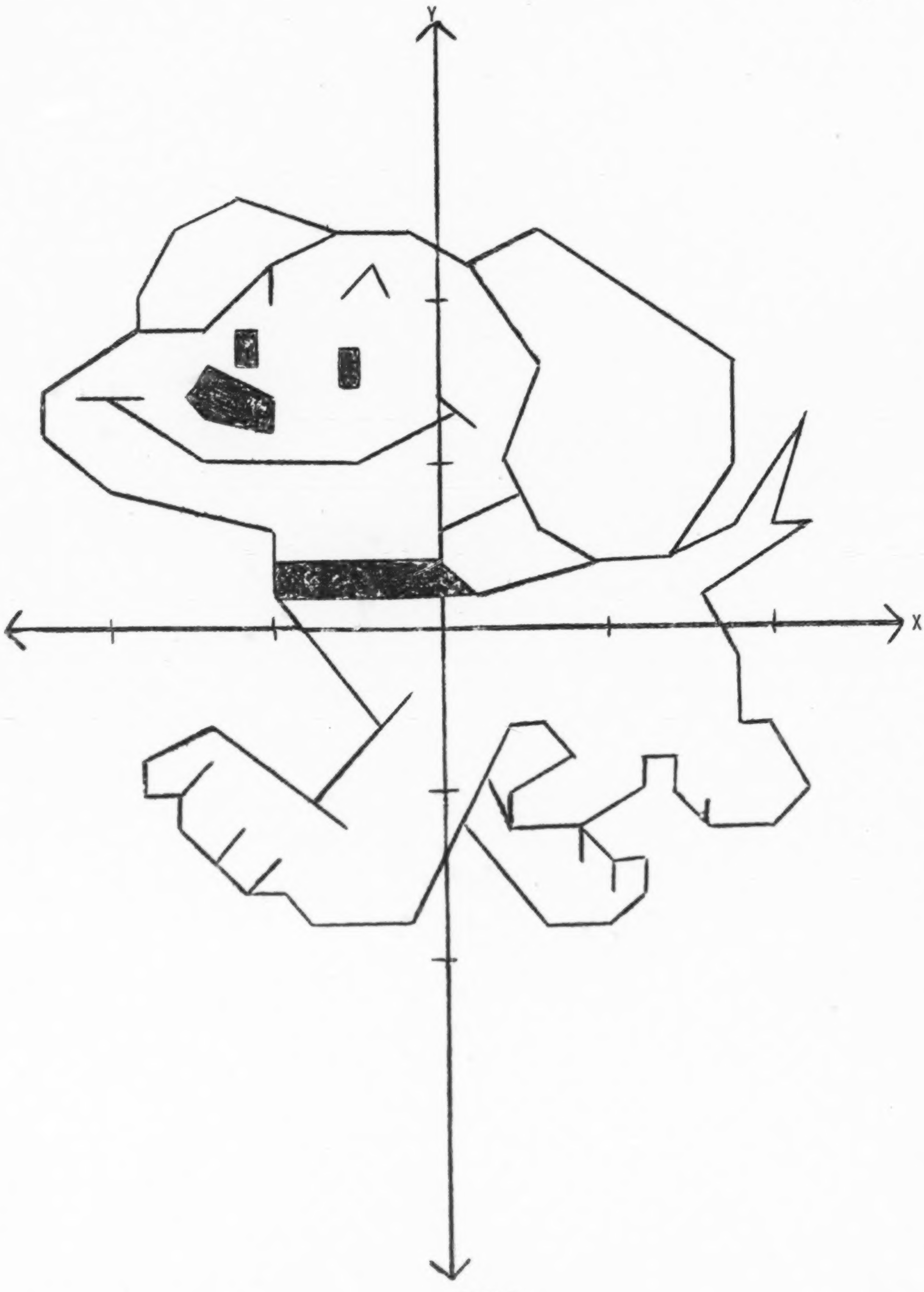
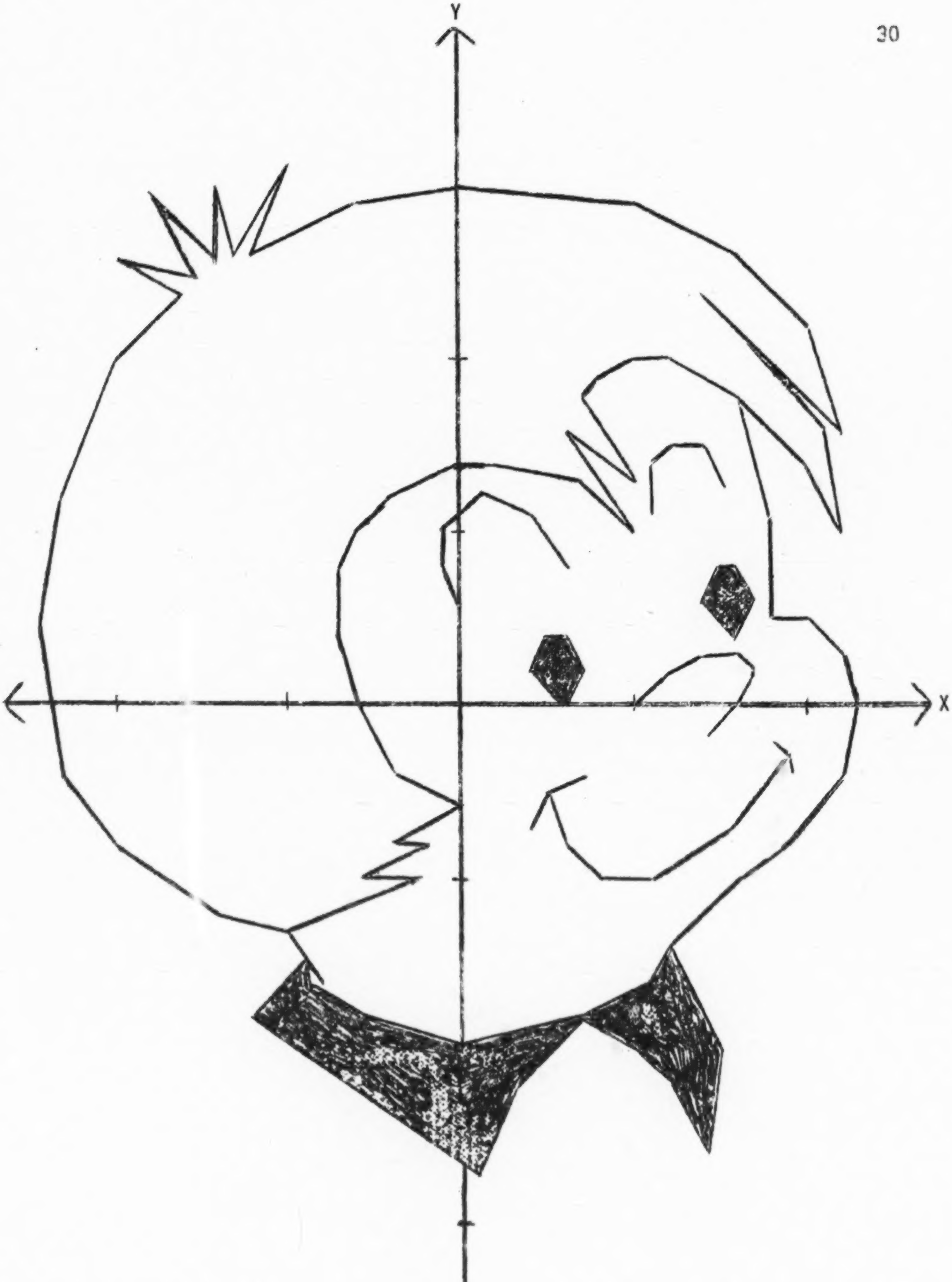


Y
X

30



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CILAMP

Central Iowa Low Achiever Mathematics Project

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Des Moines, Iowa 50311

AN INTRODUCTION TO FLOW CHARTING

BY

DAVID R. O'NEIL

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INTRODUCTION TO FLOW CHARTING

By David R. O'Neil

The following pages are devoted to the teaching of flow charting to the low-achiever in mathematics. An outline of procedures for teaching flow charting together with comments and recommendations has been included. The basic structure of the lesson plans is patterned after one developed by Terry Shoemaker of Alameda High School in Lakewood, Colorado.

PURPOSES

It is our feeling that flow charting serves several extremely useful purposes in the modern mathematics curriculum. A few of these purposes are listed below:

1) Problem Solving

Undoubtedly flow charting has its greatest application in the area of problem solving. The technique of flow charting necessitates the organizing of one's thoughts in a logical and systematic manner. The development of such a technique is of paramount importance in the solving of problems in mathematics as well as in everyday life. A flow chart is an organizational and visual aid to the understanding of a concept.

2) Use with the Calculator*

Flow charting is characteristically included in the study and use of the calculator. In order to make efficient use of the calculator, the student must be able to organize the problem which he wishes to solve.

Students receive an immediate reinforcement from correct answers obtained on the calculator. This reinforcement provides a strong motivation for them to produce the correct chart since they are required to produce the flow chart before being allowed to use the calculator. Hence, a circle of success is established using the flow chart and calculator.

3) Following Directions

An important aspect of everyday life, and of school life in particular, is the ability to follow directions. By learning to flow chart correctly, the student learns how to give directions to someone else as well as how to follow another's directions.

4) A Status Symbol

Flow charting for the low-achiever seems to give him a type of status which he enjoys. Many times he is able to brag about his ability to a "bright" peer who has not been exposed to the flow charting technique. The use of the template in flow charting also enhances the student's feeling of importance since he knows that professionals such as engineers and programmers use the template in their work.

*The calculator flow charts found in this booklet were designed to be used with the Divisuma 24 calculator (Olivetti-Underwood). Only minor changes are necessary to allow their use with other makes of calculators.

5) Reading Problems

It is possible to circumvent part of the student's reading problem via the use of flow charting. The flow chart avoids lengthy verbal instructions and is an alternative to word problems.

6) Subroutines

A valuable result of flow charting is the development of subroutines. A subroutine is that portion of the routine used to solve a problem which may be reused again in similar or related problems. A good example is the subroutine of finding the average of two or more numbers.

Once the student has successfully flow charted the routine of how to find the average of several numbers, he can use this routine over and over again in subsequent problems involving the finding of an average.

Several examples of possible subroutines follow:

(Per cent one number is of another; multiplying or dividing a number by a multiple of 10; finding the L.C.D. of G.C.F. of two or more numbers; renaming fractions as a per cent.)

SUGGESTED USES OF FLOW CHARTS

The following list of suggested uses of flow charts is by no means complete. You undoubtedly could add several uses after only working with flow charts for a short time. This list is intended only as a guide to help you get started.

1) To pre-plan how to do something

2) To reduce the complex to a simpler, more understandable form

3) To organize a problem so that it may be done in sections

4) To reduce a problem to a standard form so as to make discussion about it easier among the people concerned.

5) To add variety to traditional problem experiences

6) To evaluate the student's comprehension

CONSIDERATIONS IN THE USE OF FLOW CHARTS

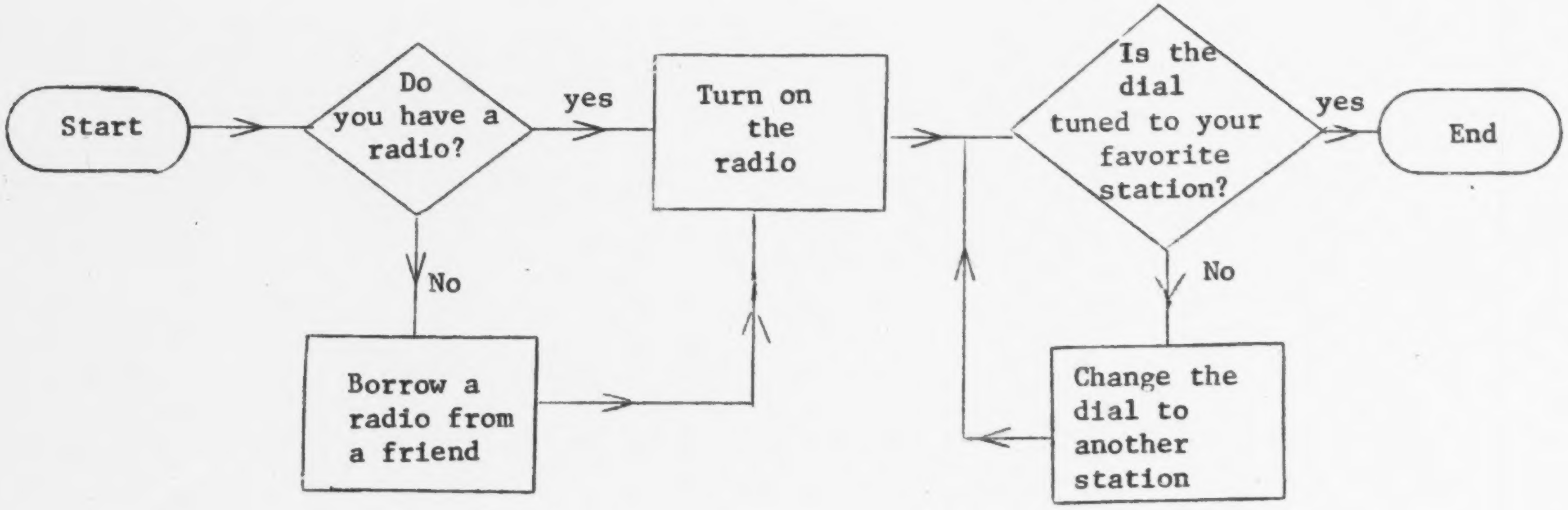
As a word of caution; DON'T overuse flow charting. It has been the experience of those teachers who have forced too much flow charting upon their students that the students soon become disenchanted with the procedure and cease to do it. Use the flow charting techniques as a tool. This tool can be used to obtain two main objectives. Namely, it provides for a high ratio of success for all of the students and serves as a vehicle to improve the logical reasoning of each student.

It has also been noted that students more readily recognize and accept flow charts which they have prepared themselves. The preparation of these flow charts may be either on an individual basis or as a class project (with teacher preparation on the chalkboard or overhead projector). Immediate use of a flow chart enhances its value and effectiveness.

An additional technique which we deem desirable is the requirement of a list of assumptions by each student before he begins his flow chart. This list of assumptions should include all of the ideas, concepts, and operations which the flow-charter assumes the reader to know and understand. By requiring such a list the students can readily see, by examining each other's flow charts, that there may be many correct flow charts for a specific problem.

These flow charts will necessarily differ because of the different assumptions. Study the two flow charts on the next pages. They both involve the tuning of a radio to your favorite station. Note that both flow charts are correct but differ because of the initial assumptions.

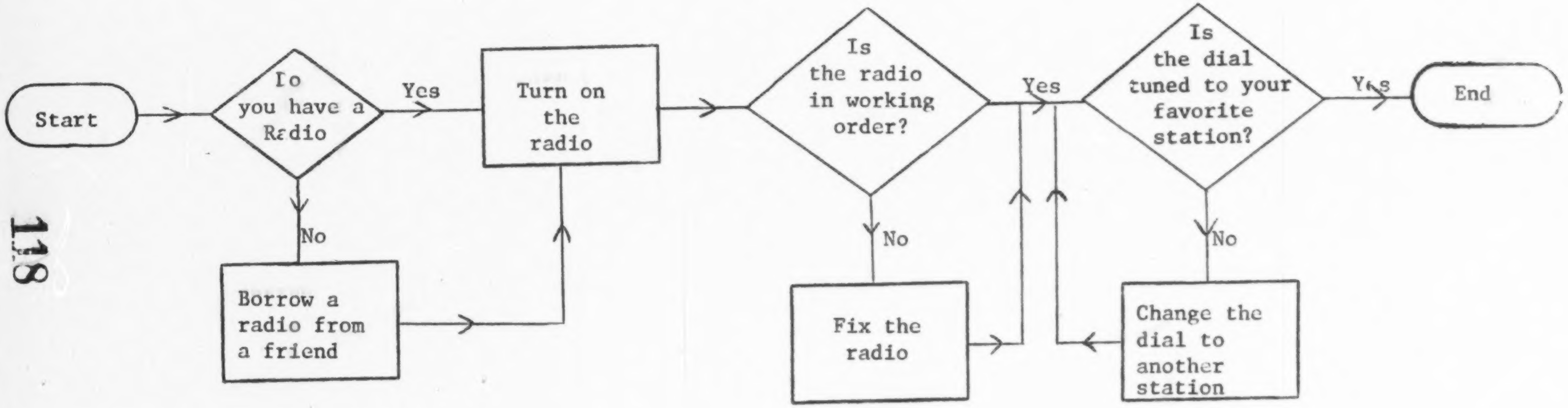
TUNING A RADIO TO YOUR FAVORITE STATION


Start
Do you have a radio?
yes
No
Borrow a radio from a friend
Turn on the radio
Is the dial tuned to your favorite station?
yes
No
Change the dial to another station
End

ASSUMPTIONS

1. You have a favorite radio station.
2. A radio is obtainable from a friend.
3. The radio is in working order.

TUNING A RADIO TO YOUR FAVORITE STATION


Start
Do you have a Radio
Yes
No
Borrow a radio from a friend
Turn on the radio
Is the radio in working order?
No
Fix the radio
Yes
Is the dial tuned to your favorite station?
No
Change the dial to another station
Yes
End

ASSUMPTIONS

1. You have a favorite radio station.
2. A radio is obtainable from a friend.
3. You are able to fix the radio if necessary.

SAMPLE FLOW CHARTS

Once your students have a knowledgeable working relationship with flow charting techniques, it is suggested that you have them try flow charting one of the four fundamental operations of arithmetic beginning with single-digit addition.

Sample flow chart(s) have been included for each of the four operations. Note that a list of assumptions has also been included. Anyone interested in pursuing additional uses of flow charts with the fundamental arithmetic operations is urged to read Experimental 9th Grade Mathematics by Groenendyk and Shoemaker. (See attached reference list)

You and/or your students may feel that some of the charts are either too precise or not precise enough. In either case, you are encouraged to discuss them and make any changes you deem necessary. One word of caution: watch the assumptions. They will need refinement if you change the charts.

It is readily agreed that you can simplify most of the charts by merely broadening the assumptions and combining some steps. In particular, the last flow chart on division can be simplified in several ways:

1) Delete the decision, "Are there more digits to the right in the dividend?" and the direction, "Division is impossible." This change would require an additional assumption that the divisor was never larger than the dividend.

2) By adding the assumption that only the remainder after the last partial quotient is to be recorded, you could eliminate several directions, "Disregard any resulting remainder."

3) By limiting the number of digits of the dividend, the length of the flow chart could be reduced.

It should prove worthwhile to discuss with your students the possibility of making another flow chart of the same operation (Tuning the Radio). Look for questions or comments like, "What would happen if you know how to fix the radio?" or "Suppose you had no friends who had a radio which you could borrow." It is important to note that nearly all flow charts may be put on a higher, more complex level by restricting the assumptions. Restricting the assumptions usually requires additional subroutines.

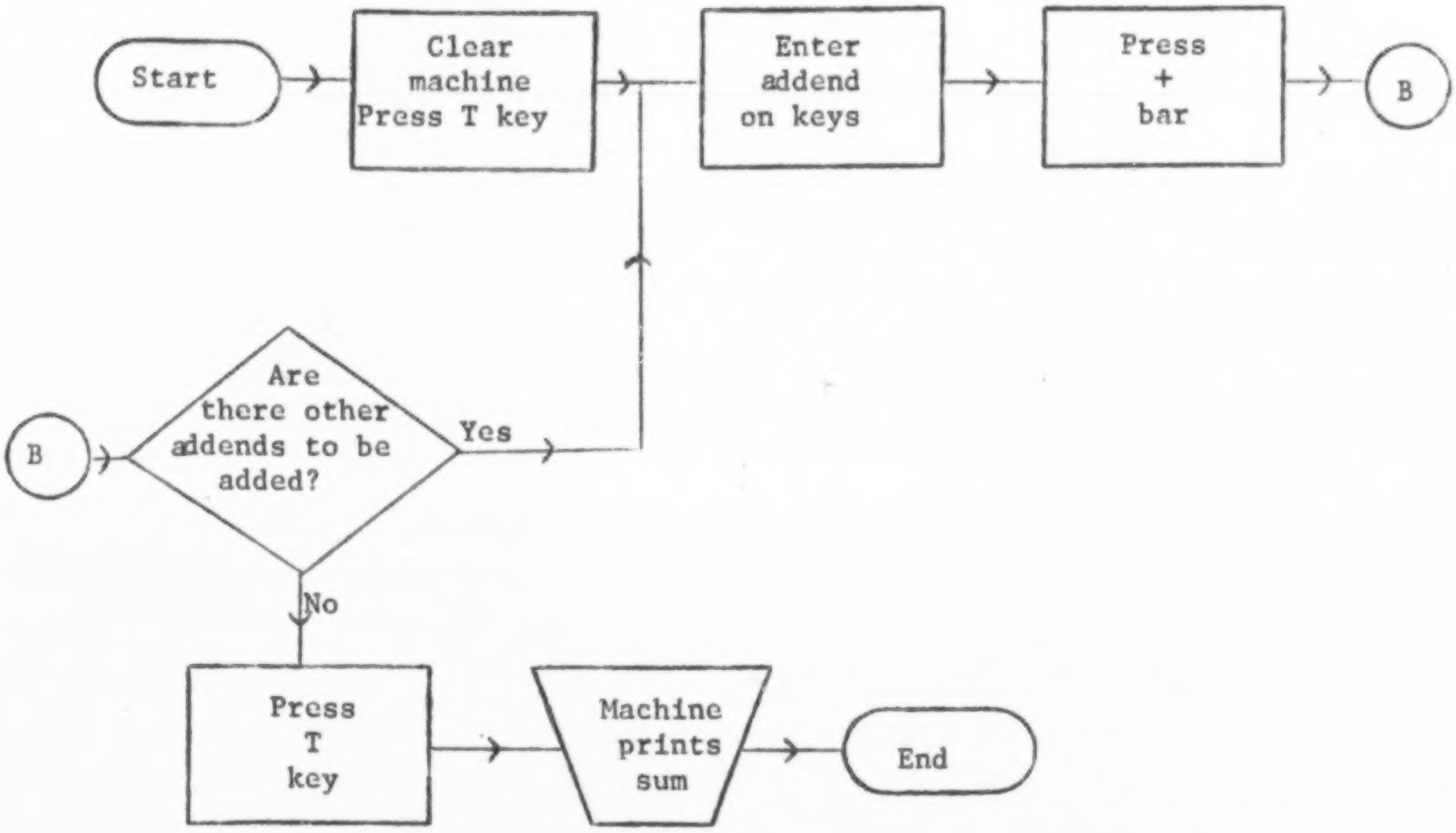
Some advantages of requiring the listing of assumptions are indicated in the recommendations prior to lesson three in the accompanying outline. Listing the assumptions enables you to better understand the reasoning, logical or not, of your students. It also allows for more than one correct answer. This fact alone is a real boon to the low-achiever. For the first time, in many cases, he is not under pressure to produce THE RIGHT ANSWER.

An integral part of flow charting is the impossibility of including all eventualities in the chart. One must, out of necessity, make certain basic assumptions before beginning work on a flow chart. It is these different assumptions which lead to correct but different flow charts for the same problem.

FLOW CHARTS FOR USE WITH CALCULATORS

The following four pages contain examples of flow charts for performing the fundamental operations with whole numbers on the calculator. In each case, the assumptions preceeding the use of the calculator have been listed.

CALCULATOR ADDITION FLOW CHART

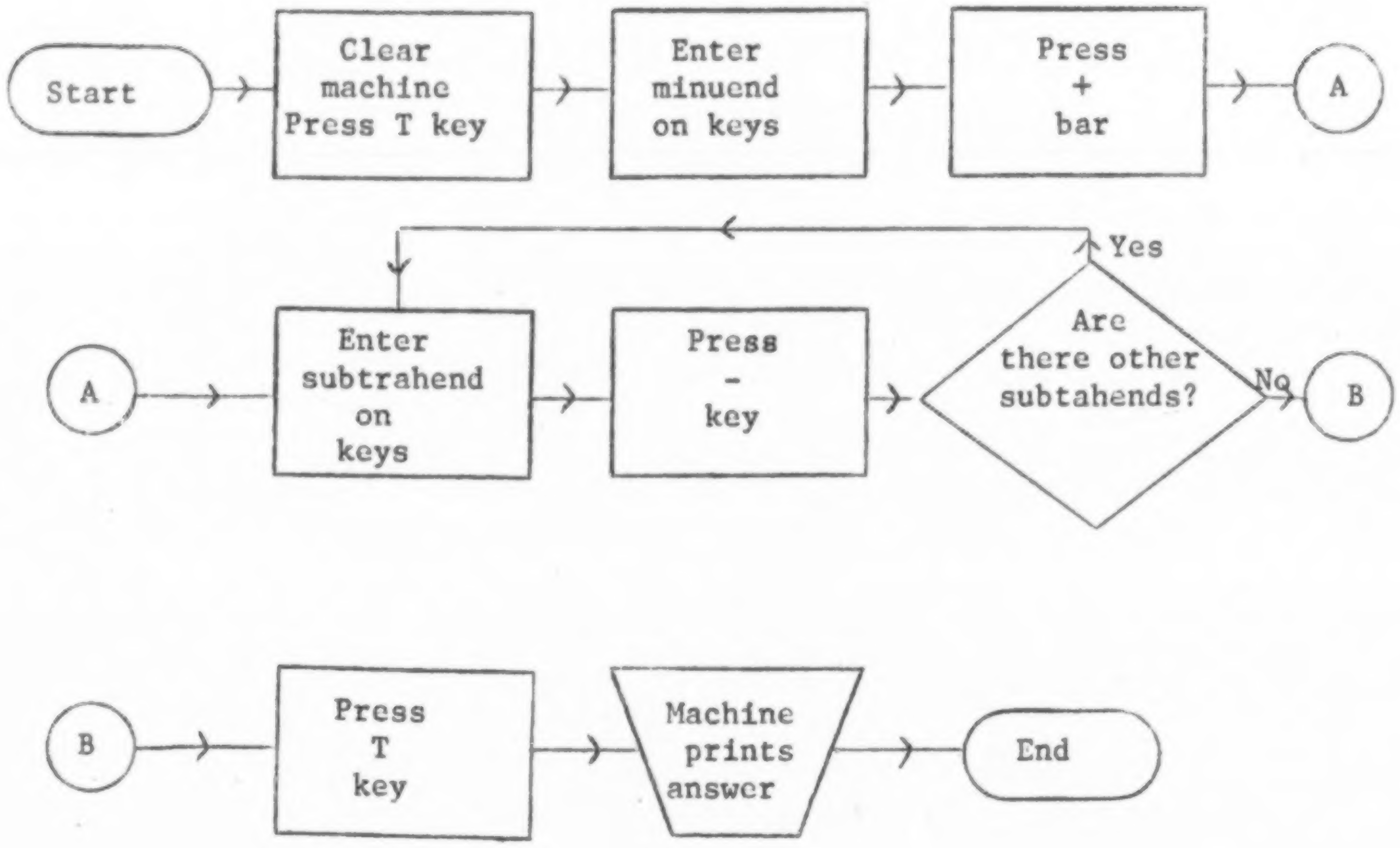

Start
Clear machine Press T key
Enter addend on keys
Press + bar
B
B
Are there other addends to be added?
Yes
No
Press T key
Machine prints sum
End

ASSUMPTIONS

1. The student knows how to turn on the calculator and the names of the keys.
2. The student knows what an addend is.
3. All numbers used are integers.
4. No negative values are to be added.*

*If there is a need to add negative values, they must be entered on the keys and followed by depressing the red - key. (Follow the subtraction flow chart.)

CALCULATOR SUBTRACTION FLOW CHART


Start
Clear machine Press T key
Enter minuend on keys
Press + bar
A
Yes
A
Enter subtrahend on keys
Press - key
Are there other subtahends?
No
B
B
Press T key
Machine prints answer
End

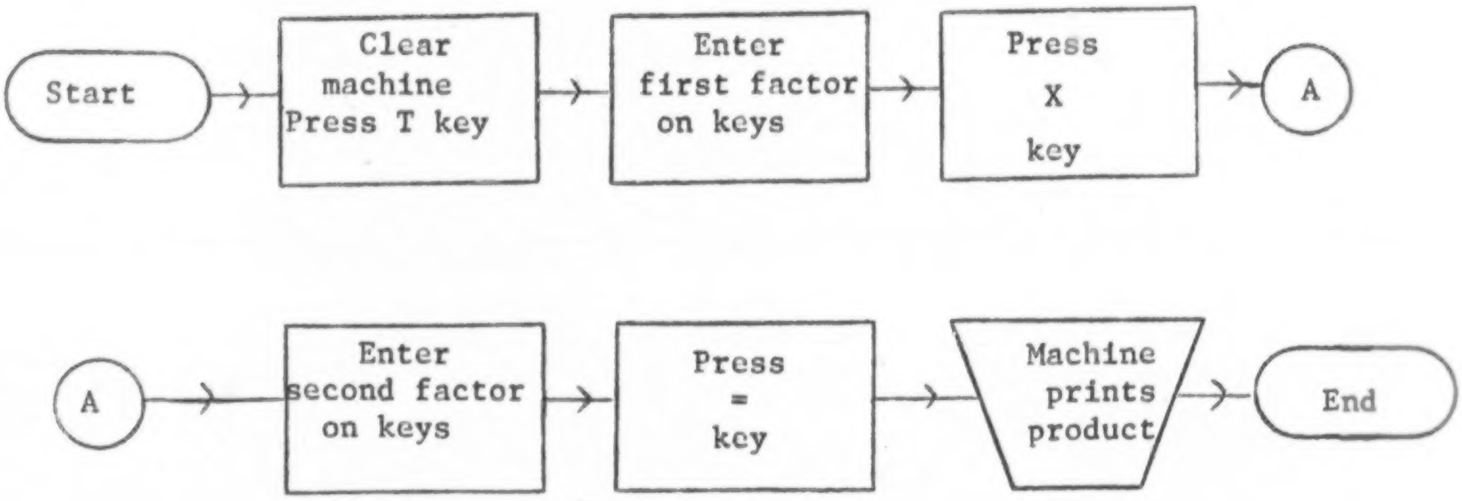
ASSUMPTIONS

1. The student knows how to turn on the calculator and knows how to operate the keys.
2. The student knows what a minuend and a subtrahend are.
3. All numbers are integers.
4. No negative values are to be subtracted.*
5. The student recognizes the postscrip "C" denotes a negative number.

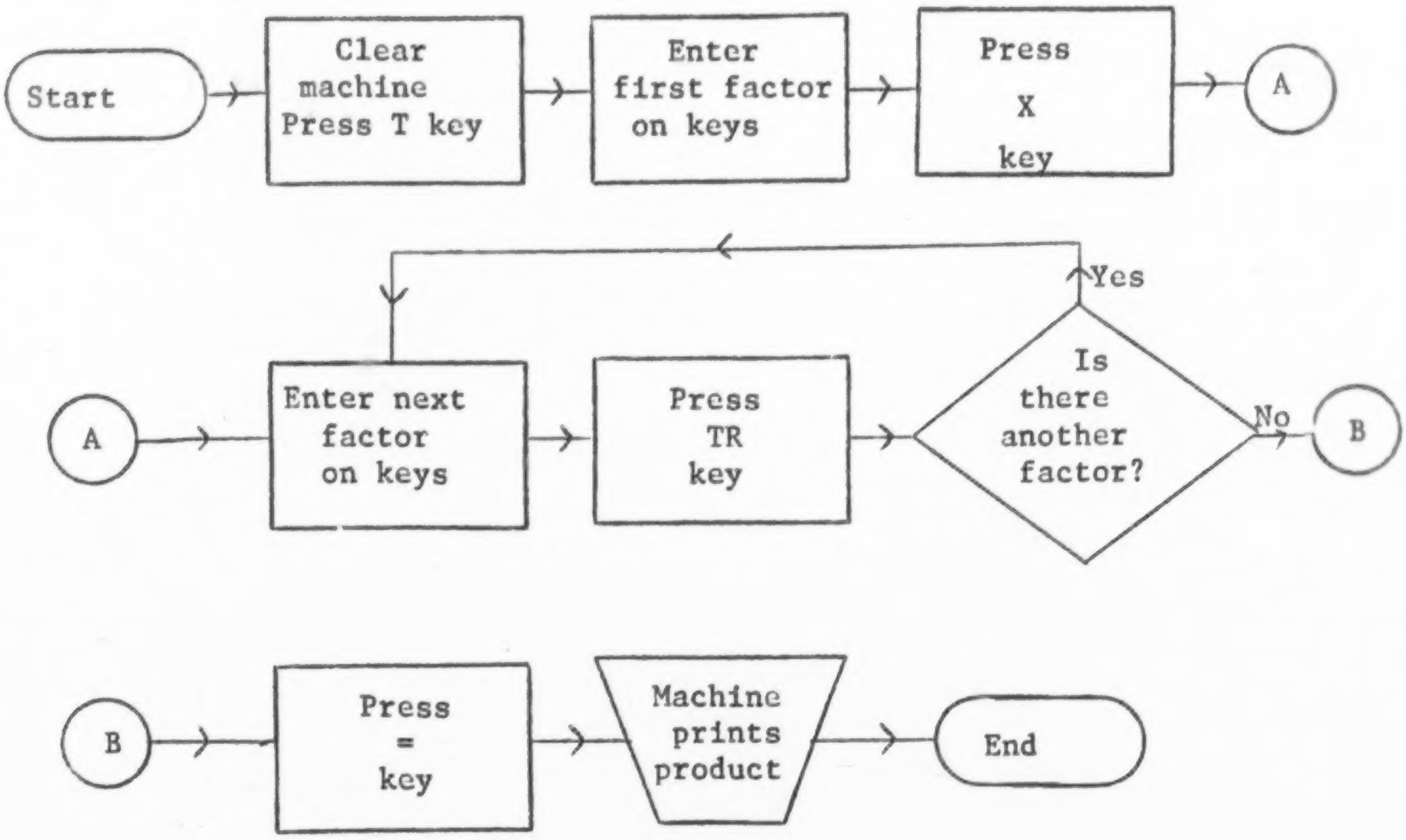
*If a negative value is to be subtracted, add it to the previous number. (See addition flow chart.)

CALCULATOR MULTIPLICATION FLOW CHARTS

(two factors)


Start
Clear machine Press T key
Enter first factor on keys
Press X key
A
A
Enter second factor on keys
Press = key
Machine prints product
End

(more than two factors)

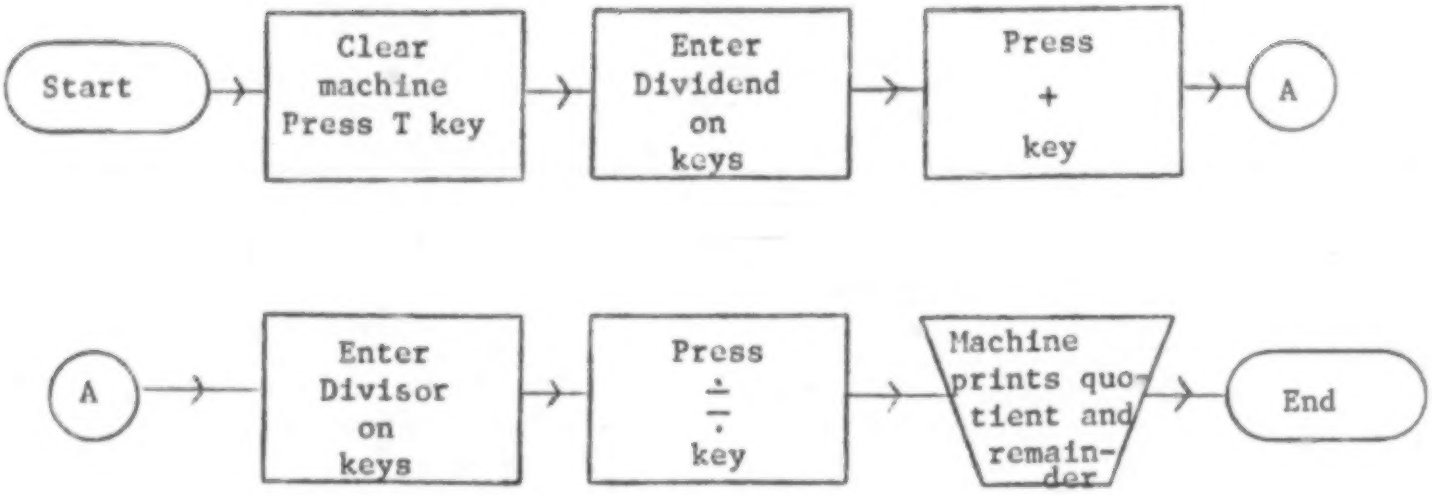

Start
Clear machine Press T key
Enter first factor on keys
Press X key
A
A
Enter next factor on keys
Press TR key
Is there another factor?
Yes
No
B
B
Press = key
Machine prints product
End

ASSUMPTIONS

1. The student knows how to operate the calculator keys.
2. Students know what factors are.
3. All numbers used are integers.
4. No negative numbers are to be multiplied*

*If negative number is to be multiplied, depress red X key.

CALCULATOR DIVISION FLOW CHART


Start
Clear machine Press T key
Enter Dividend on keys
Press + key
A
A
Enter Divisor on keys
Press ÷ key
Machine prints quotient and remainder
End

ASSUMPTIONS

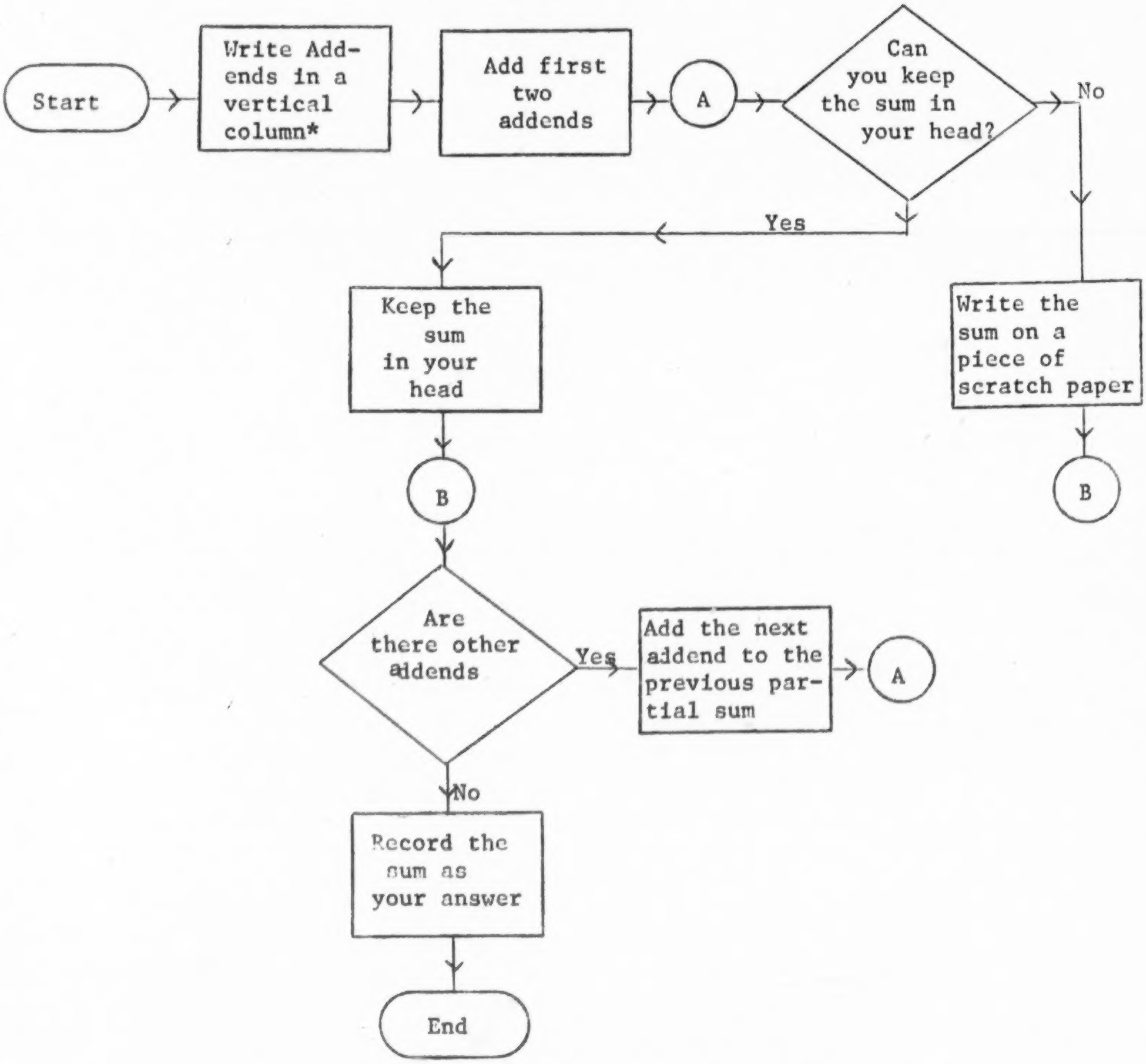
1. Students know how to operate the calculator keys.
2. Students know what dividend, divisor, quotient, and remainder are.
3. All values used are positive integers.
4. No negative numbers are to be used.

OPERATIONS FLOW CHARTS

The following six examples are of flow charts for the four fundamental operations without the aid of a calculator. You will note that the charts are more complex because of the necessity of instructing the reader as to physical placement of digits. This was unnecessary in the previous charts since the machine automatically compensated for digit placement. Again, the assumptions have been listed for each chart.

ADDITION FLOW CHART

(Single-Digit Addends)


Start
Write Addends in a vertical column*
Add first two addends
A
Can you keep the sum in your head?
No
Yes
Keep the sum in your head
Write the sum on a piece of scratch paper
B
B
Are there other addends
Yes
Add the next addend to the previous partial sum
A
No
Record the sum as your answer
End

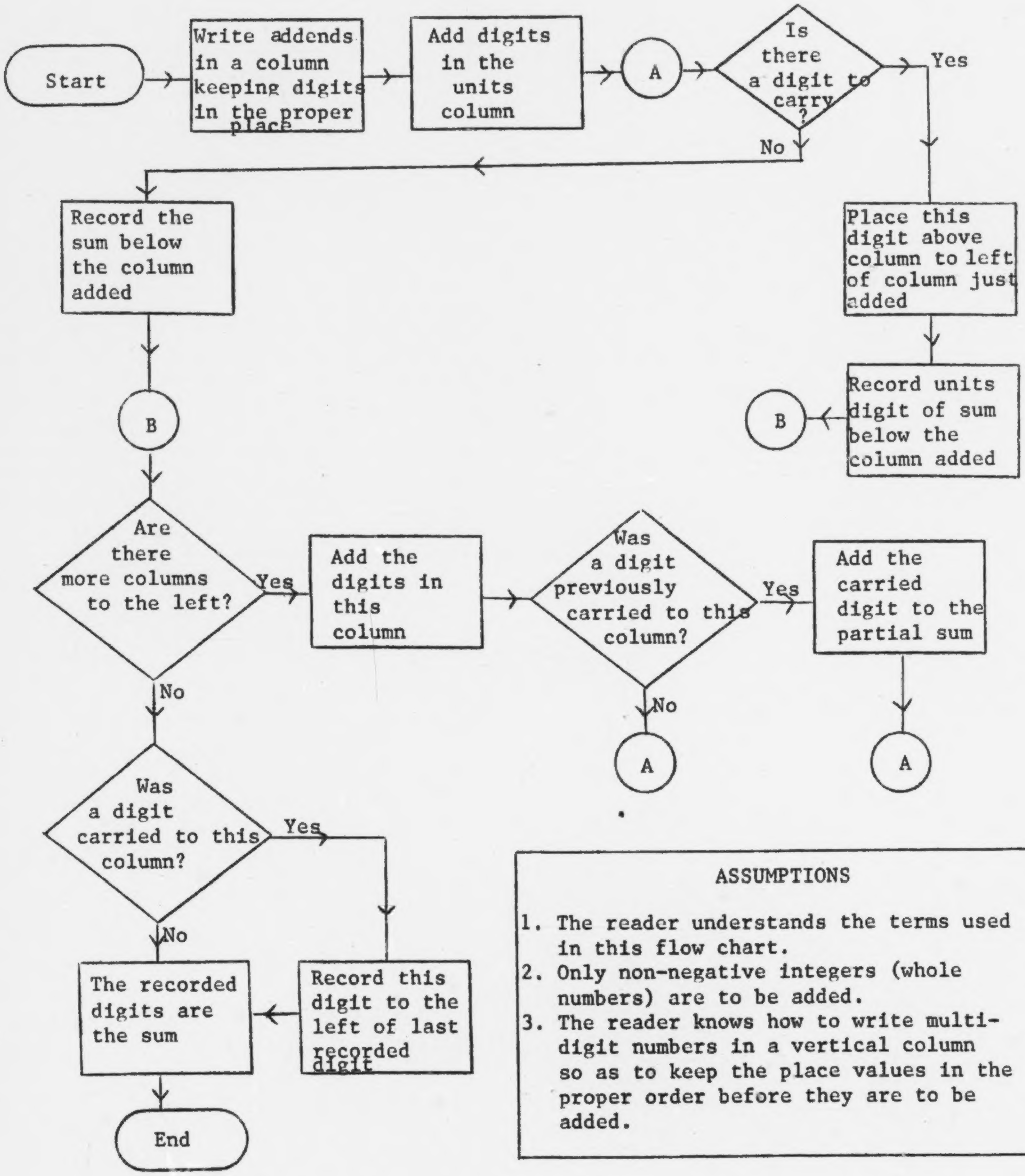
ASSUMPTIONS

1. Student knows what addend, vertical column, and partial sum are.
2. Only positive integers (whole numbers) are to be added.

*You may desire to place addends horizontally. Add from left to right so left two addends are added first.

ADDITION FLOW CHART

(Multiple-Digit Addends)

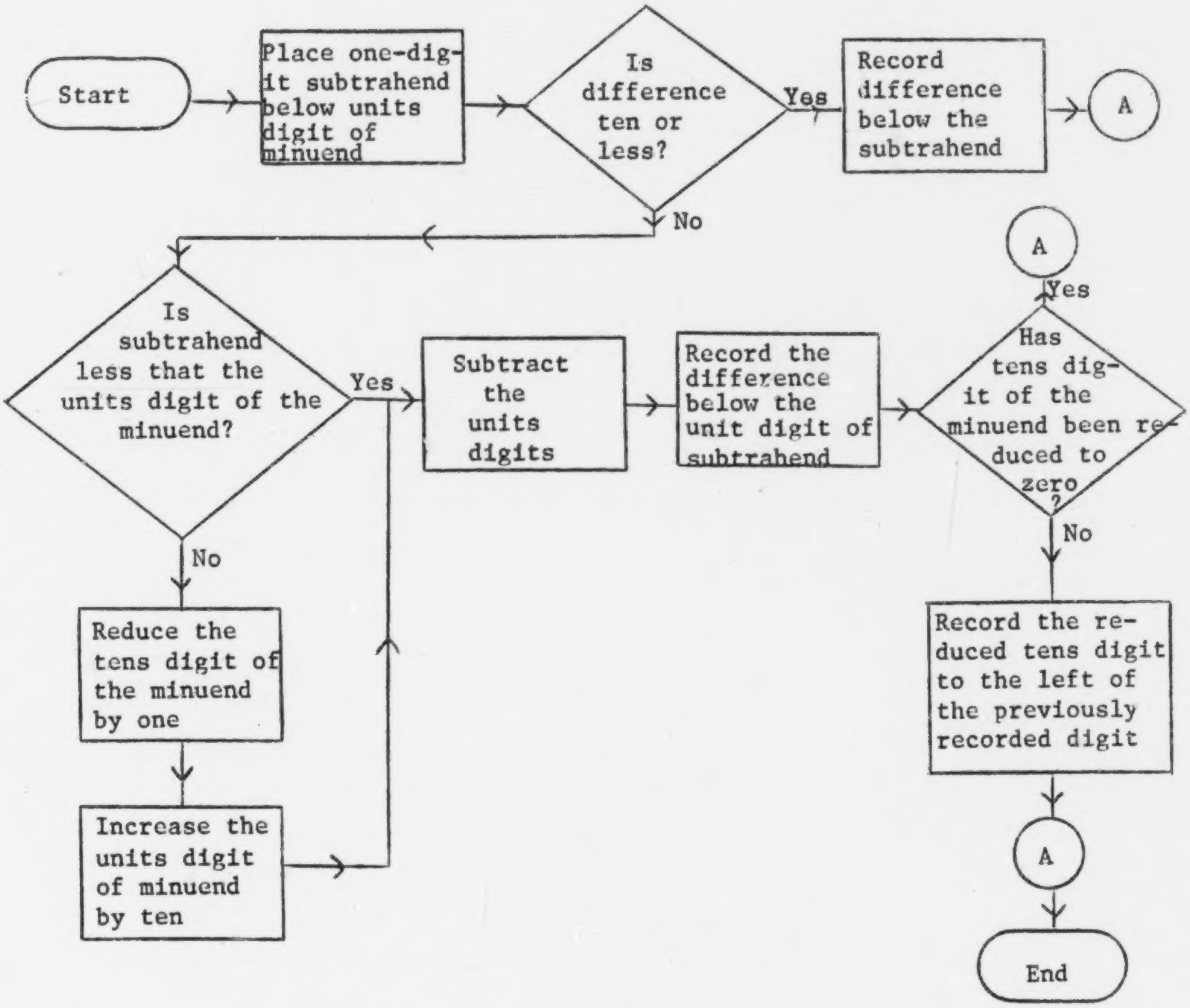

Start
Write addends in a column keeping digits in the proper place
Add digits in the units column
A
Is there a digit to carry ?
Yes
No
Place this digit above column to left of column just added
Record units digit of sum below the column added
B
Record the sum below the column added
B
Are there more columns to the left?
Yes
No
Add the digits in this column
Was a digit previously carried to this column?
Yes
No
Add the carried digit to the partial sum
A
A
Was a digit carried to this column?
Yes
No
Record this digit to the left of last recorded digit
The recorded digits are the sum
End

ASSUMPTIONS

1. The reader understands the terms used in this flow chart.
2. Only non-negative integers (whole numbers) are to be added.
3. The reader knows how to write multi-digit numbers in a vertical column so as to keep the place values in the proper order before they are to be added.

SUBTRACTION FLOW CHART

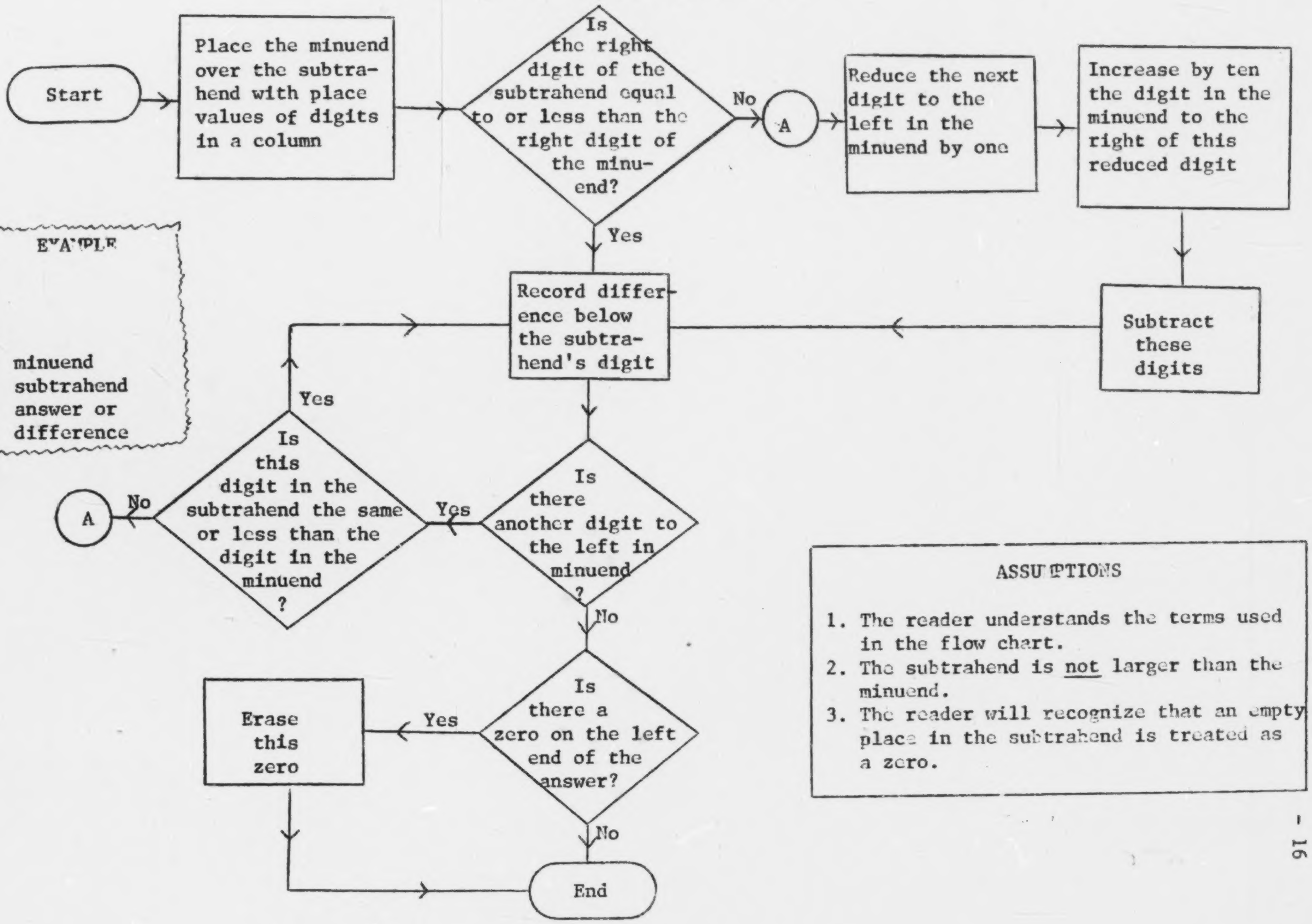
(One-Digit from Two-Digit number)


Start
Place one-digit subtrahend below units digit of minuend
Is difference ten or less?
Yes
No
Record difference below the subtrahend
A
Is subtrahend less that the units digit of the minuend?
Yes
No
Reduce the tens digit of the minuend by one
Increase the units digit of minuend by ten
Subtract the units digits
Record the difference below the unit digit of subtrahend
Has tens digit of the minuend been reduced to zero?
Yes
No
A
Record the reduced tens digit to the left of the previously recorded digit
A
End

ASSUMPTIONS

1. The reader understands the terms used in this flow chart.
2. The subtrahend is not larger than the minuend which assures a non-negative difference.
3. The reader is able to determine all differences between two numbers that differ by ten or less.

SUBTRACTION FLOW CHART
(More than One-Digit numbers)


Start
Place the minuend over the subtrahend with place values of digits in a column
Is the right digit of the subtrahend equal to or less than the right digit of the minuend?
No
A
Reduce the next digit to the left in the minuend by one
Increase by ten the digit in the minuend to the right of this reduced digit
Subtract these digits
Yes
Record difference below the subtrahend's digit
Is there another digit to the left in minuend ?
Yes
Is this digit in the subtrahend the same or less than the digit in the minuend ?
Yes
No
A
No
Is there a zero on the left end of the answer?
Yes
Erase this zero
No
End

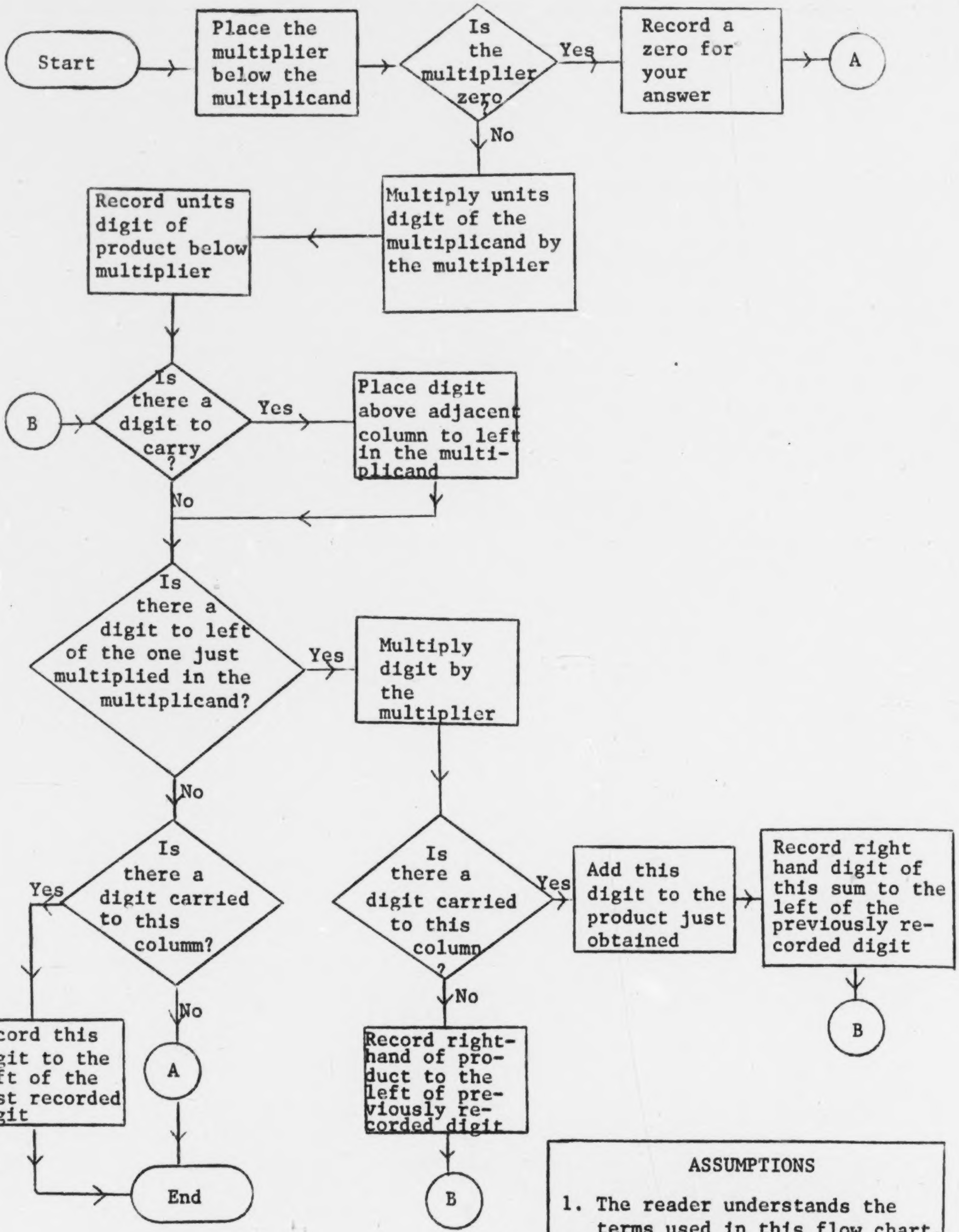
EXAMPLE

hundreds	tens	units	
5	3	6	minuend
	8	7	subtrahend
4	4	9	answer or difference

ASSUMPTIONS

1. The reader understands the terms used in the flow chart.
2. The subtrahend is not larger than the minuend.
3. The reader will recognize that an empty place in the subtrahend is treated as a zero.

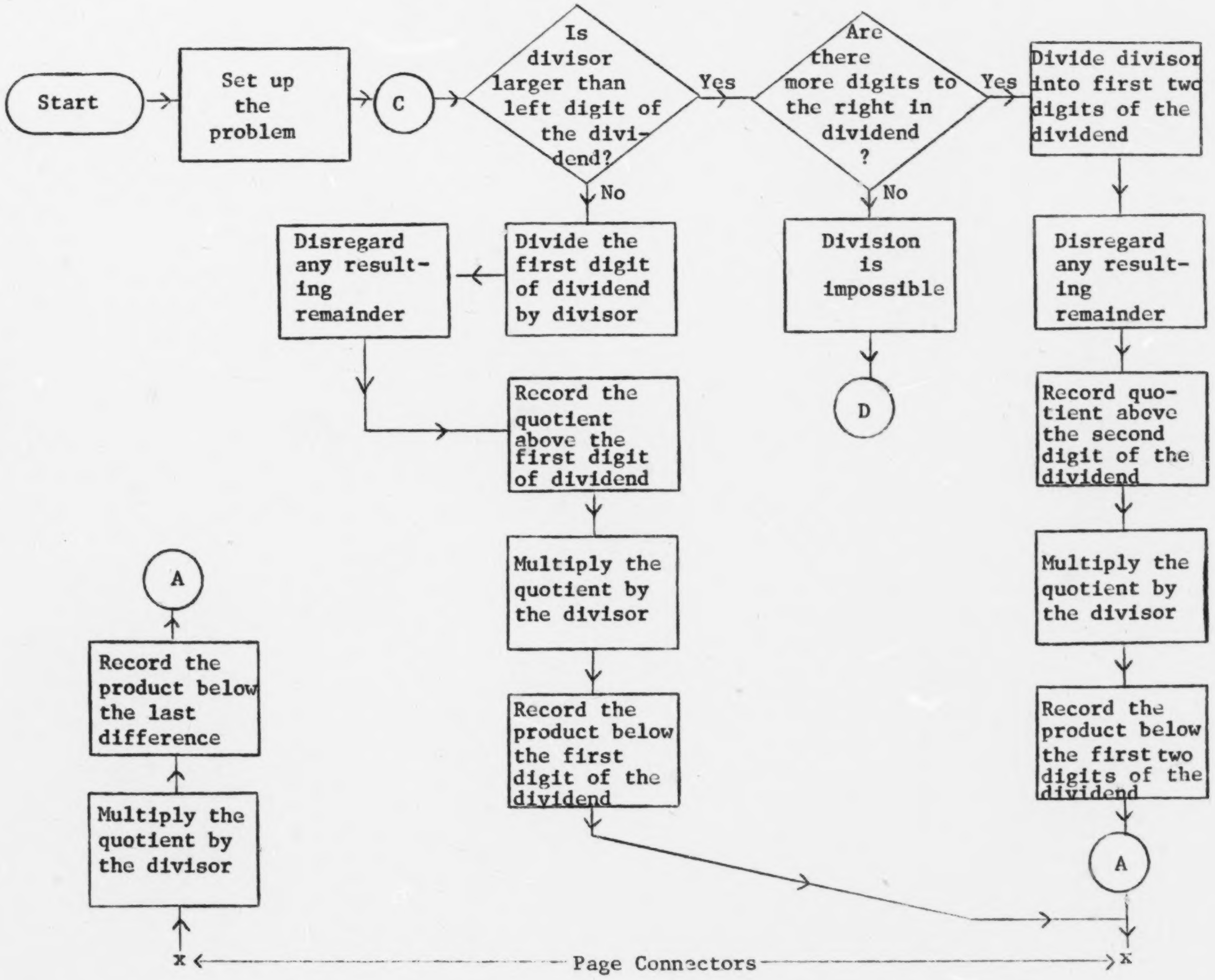
ONE-DIGIT MULTIPLIER FLOW CHART

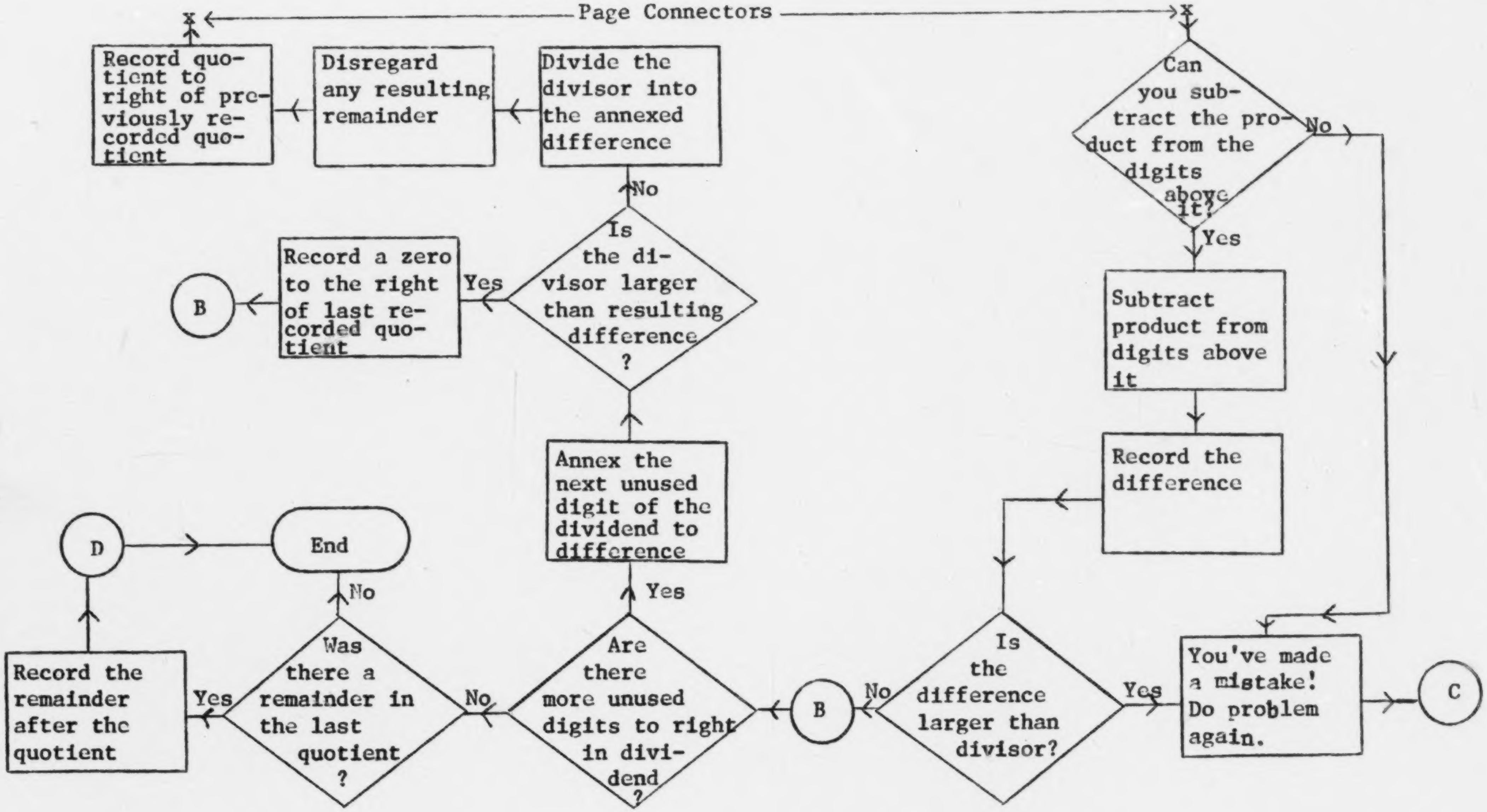

Start
Place the multiplier below the multiplicand
Is the multiplier zero?
Yes
Record a zero for your answer
A
No
Multiply units digit of the multiplicand by the multiplier
Record units digit of product below multiplier
B
Is there a digit to carry?
Yes
Place digit above adjacent column to left in the multiplicand
No
Is there a digit to left of the one just multiplied in the multiplicand?
Yes
Multiply digit by the multiplier
No
Is there a digit carried to this columm?
Yes
Record this digit to the left of the last recorded digit
No
A
End
Is there a digit carried to this column?
Yes
Add this digit to the product just obtained
Record right hand digit of this sum to the left of the previously recorded digit
B
No
Record right-hand of product to the left of previously recorded digit
B

ASSUMPTIONS

1. The reader understands the terms used in this flow chart.
2. Only non-negative integers (whole numbers) are to be multiplied.

DIVISION FLOW CHART
(Single-Digit Divisor)


Start
Set up the problem
C
Is divisor larger than left digit of the dividend?
Yes
No
Are there more digits to the right in dividend ?
Yes
No
Divide divisor into first two digits of the dividend
Divide the first digit of dividend by divisor
Disregard any resulting remainder
Division is impossible
D
Disregard any resulting remainder
Record the quotient above the first digit of dividend
Record quotient above the second digit of the dividend
Multiply the quotient by the divisor
Multiply the quotient by the divisor
Record the product below the first digit of the dividend
Record the product below the first two digits of the dividend
A
A
Record the product below the last difference
Multiply the quotient by the divisor
x
Page Connectors
x


Page Connectors
x
x
Can you subtract the product from the digits above it?
No
Yes
Subtract product from digits above it
Record the difference
Is the difference larger than divisor?
Yes
No
You've made a mistake! Do problem again.
C
B
Are there more unused digits to right in dividend ?
Yes
No
Annex the next unused digit of the dividend to difference
Is the divisor larger than resulting difference ?
No
Yes
Divide the divisor into the annexed difference
Disregard any resulting remainder
Record quotient to right of previously recorded quotient
Record a zero to the right of last recorded quotient
B
Was there a remainder in the last quotient ?
No
Yes
End
Record the remainder after the quotient
D

ASSUMPTIONS

1. The reader knows how to record a remainder.
2. The reader understands that digits are numbered from left to right in this flow chart.
3. The reader understands the terms.
4. The reader knows how to "set up" division problems.
5. The reader knows basic facts through ten for the fundamental operations.

OUTLINE OF LESSONS ON FLOW CHARTING

It should be noted that, although the following outline is divided into lessons, this does not imply that the material outlined is necessarily to be covered in one day. You may find that the students fail to understand a concept and therefore need more time in which to grasp the idea. Conversely, you might observe a ready assimilation of the material to the point that two lessons could be combined into one. You must be prepared for either case.

LESSON ONE

FLOW CHARTING

Introduction to Flow Charting

1. Definition and applications.
2. Illustration with concrete examples such as: Using the Telephone; Filling the Car with Gas; etc.
3. Discussion on the complexity of a flow chart.
4. Describe the six basic symbols used in flow charting.

Practice in Preparation of Flow Charts

1. Assign another situation encountered daily by all members of the class such as: Coming to School; Getting Ready for Bed; etc.
2. Other assigned concrete problems on flow charting as time permits and dependent on class comprehension.

Recommendations

Although flow chart templates are available they are not essential for the beginning flow-charter to use in understanding the concept.

Sometime during the first class session on flow charting the instructor should point out the depth and complexity which any situation can provide, depending upon the depth of knowledge and desires of the programmer.

The first flow chart (a concrete situation) should be a situation familiar to all of the students.

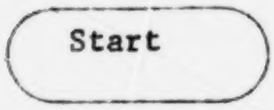
Some examples of flow charts that could be assigned for the second class session include:

Going to Work
Calling Long Distance (Direct Distance Dialing)
Getting Up in the Morning
Opening a Door

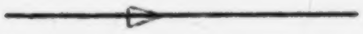
LESSON ONE (continued)

An example of each of the six basic symbols mentioned previously are listed below. Note that the final symbol (6) is used in conjunction with calculator printout.

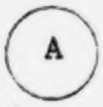
1) Terminal (start or stop)



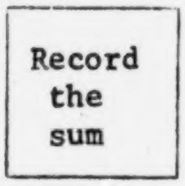
2) Direction Arrows



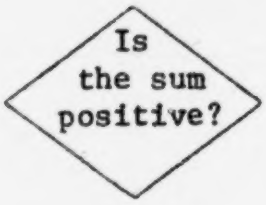
3) Connectors



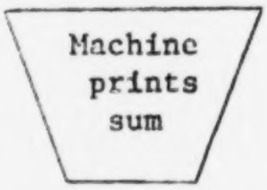
4) Instruction


Record
the
sum

5) Decision


Is
the sum
positive?

6) Printed Output


Machine
prints
sum

LESSON TWO

FLOW CHARTING

Review of Basic Symbols Used in Flow Charting

1. The uses and misconceptions of using the symbols.
2. Observation of flow charts from assignment -- put on chalkboard or transparencies.
3. Constructive criticism of the displayed flow charts.

Extension of the Flow Chart Concept -- Building Charts of Semi-Abstract Situations

1. Prepare, with the assistance of the students, a flow chart of a semi-abstract situation such as:
   a. Averaging a Column of Integers.
   b. Adding Two Simple Fractions.
2. Explain the loop.
3. Assign the problem of flow charting the multiplication of two numbers.

Recommendations

When criticizing the assigned work from the past session, be sure the students do most of the discussing and thus show you where they have had difficulty comprehending the material previously presented. Be sure the criticism is constructive.

In the discussion of the mutually constructed flow chart, be sure the class understands there are a number of assumptions made and what they are, i.e. the person is assuming whoever uses the flow chart knows how to add, subtract, etc. In the assignment for lesson three, request the students to list their assumptions before beginning work on the flow chart. This will enable you to observe the student's ability to think logically. In addition, it forces the student to preface his work with a form of introduction or explanation to the eventual reader, telling the reader what he is expected to know before he can perform the task indicated by the flow chart.

LESSON THREE

FLOW CHARTING

Review of and Questions about Assignment Two

1. Question and answer session.
2. Other remarks.

Semi-Abstract Situations for Flow Charting in Depth

1. Assign each student the task of making a flow chart that could be used to to show the commutative and associative properties in mathematics.
2. Assign a conversion problem of some type, i.e. minutes to hours, ounces to pounds, etc.

Recommendations

Students should continue to list all assumptions they make. In preparing the flow chart for the associative and commutative properties, the students will have to have an explanation as to the basic ideas behind the development of such properties.

The conversion assignment can be from a situation in business and industry if the instructor so desires. However, it should be practical.

LESSON FOUR

FLOW CHARTING

Discussion of the assigned work from previous session

1. Be sure the attitudes students develop will enable them to build successful flow charts.
2. They should realize what the conversion factor is in the last portion of the previously assigned flow chart.
3. Reassign the same type of lesson again if there is still apprehension about the previous assignment.

Preparing for Programming Flow Charting

1. Assign and allow time in the class to assist the students in preparing a flow chart that includes a comparison. Perhaps two flow charts would be even better with the first a more recognizable comparison situation. The first one might be, "Adding many several digit numbers" followed by "How to Compute a Man's Pay." (Considering dependents, FICA, and State and Federal Withholding Tax.)
2. Build a feeling of confidence in the student to logically illustrate his thinking. This can be done if the problems he is asked to solve are not too difficult for him, but at the same time, have meaning and appeal to him.

Recommendations

The instructor may want to display the previous session's assignment on the chalkboard or overhead projector and discuss the logical flow of those displayed. A discussion of the conversion and comparison is not out of order at the beginning of the session to build for future work.

Allow a considerable amount of time for students to work on the assignment for the next session. Ample time will foster a lesser amount of confusion and frustration.

LESSON FIVE

FLOW CHARTING

The entire fifth session, if needed at all, would be to clear up any misgivings and lack of comprehension of what a flow chart is and the logical deduction that is taking place.

Recommendations

Apply the flow charting done this session to mathematical situations where the student prepares a chart as well as solving the problem.

SUGGESTED DOS AND DON'TS OF FLOW CHARTING

A list of suggested DOS and DON'TS has been included for your use and comments. Again, this list is not meant to be a complete compilation. It is merely a collection of some ideas which the author feels should be important to you as a teacher. You are encouraged to comment on those items listed as well as add to the existing list.

DOS

1) Encourage students to re-use subroutines once they have been flow-charted.
2) Give students a choice of flow chart exercises by providing a list of possibilities to choose from.
3) Make the problems as "real life" as possible.
4) As with any procedure involving low-achievers, attempt to choose problems so each child may have success solving them.
5) Make sure the student understands the problem before he attempts to flow chart it.
6) Use the motivational factor of flow charting to introduce new mathematical terms.
7) Provide a place or means of storing student's flow charts.
8) Have available for classroom use several transparencies of flow charts. Allow students to put their work on the overhead.
9) Devote a bulletin board to flow charting sometime during the study of flow charts.
10) Encourage students to write down assumptions before beginning any flow chart.

DON'TS

1) Avoid re-doing the same chart over and over.
2) Avoid letting the flow chart problem become too complex.
3) Avoid over-use of the flow chart.
4) Avoid being over critical of students whose first efforts at flow charting fail. Their logic may be much different from yours.

Comments should be directed to: Mr. David O'Neil, Coordinator, Central Iowa Low-Achiever Mathematics Project, 1164 26th Street, Des Moines, 50311.

FLOW CHARTING REFERENCES

Experimental 9th Grade Mathematics, Groenendyk and Shoemaker, Central University of Iowa Press, Pella, Iowa 50219.

Both teacher's and student's manuals that employ flow charting and "real life" problems.

Explanations Based on Structural Principles of Mathematics, Joe L. Lancaster, Secondary Mathematics Consultant, Dallas Independent School District, Dallas, Texas 75204.

Uses flow charting and printing calculator as basic teaching devices to explain and illustrate structural principles of mathematics.

Flow Charting the Logical Processes of Mathematics, Joseph Lieberman, Supervisor of Mathematics, Levittown Public Schools, North Village Green, Levittown, New York 11756.

A set of flow charts that teach general mathematics using the calculator.

Involving Low-Achievers in Mathematics, Terry Shoemaker, Alameda High School, 1255 S. Wadsworth, Lakewood, Colorado 80226

Explains how to set up mathematics laboratory, use flow charting and experiments in general mathematics classes.

Low-Achiever Motivational Program (LAMP), A. Wilson Goodwin, Supervisor of Mathematics, Des Moines Public Schools, 1800 Grand Avenue, Des Moines, Iowa 50307

A booklet that contains examples of flow charting "real life" problems from industry and commerce, enrichment projects, and other high involvement materials for use in general mathematics classes.

Midland Schools, Iowa State Education Association, May, 1964, pp 10-15,18.

A series of articles by Eldert A. Groenendyk including Lab approaches to general math, calculators in mathematics, and flow charting.

Olivetti Underwood Divisumma 24 Flowcharting Manual, Olivetti Underwood Corp., One Park Avenue, New York, New York 10016. Cost $1.25

Explains how to use the Divisumma 24 Calculator. Templates may be obtained from this company.

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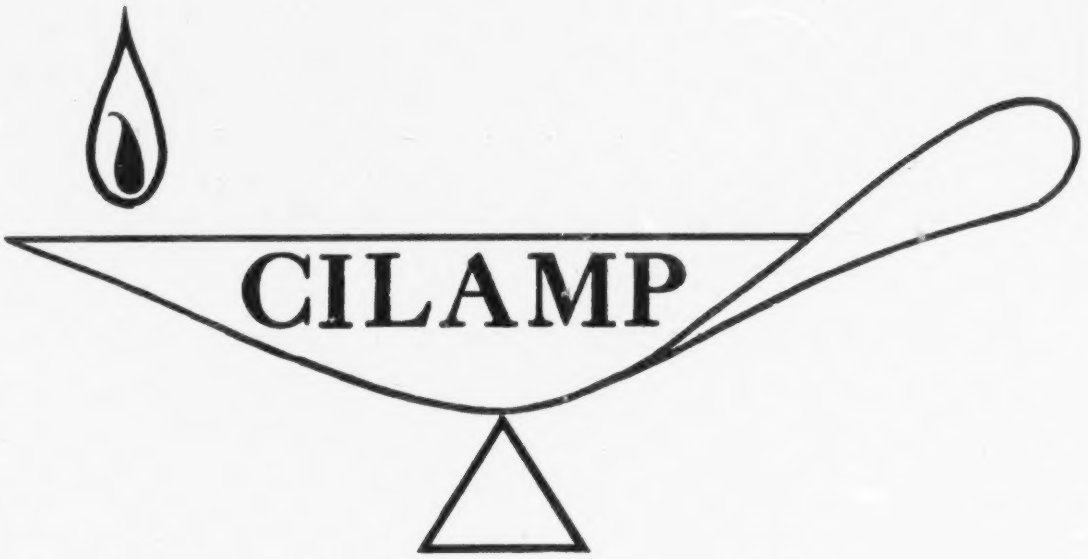

Central Iowa Low Achiever Mathematics Project

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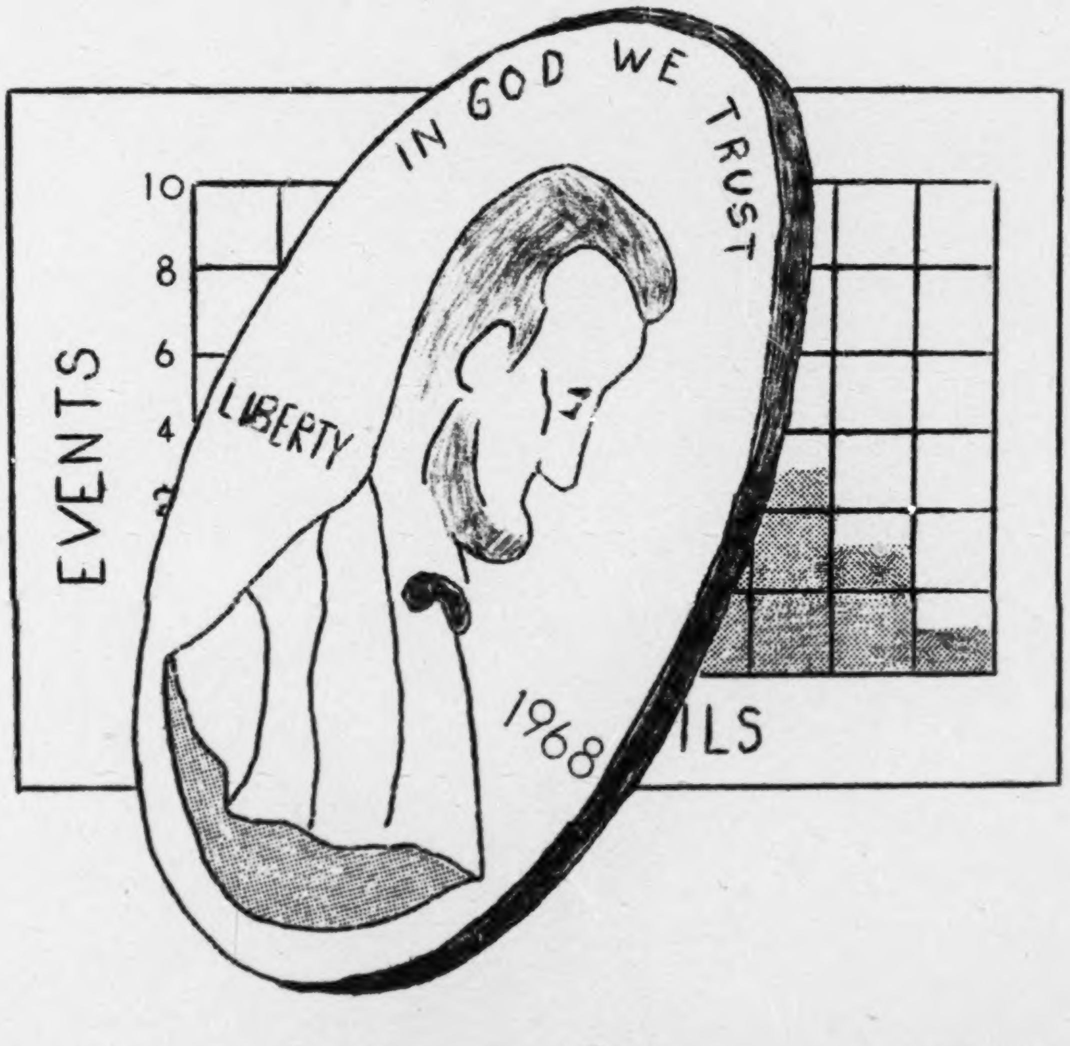
FIRST PROBABILITY PROGRAM


IN GOD WE TRUST
LIBERTY
1968
10
8
6
4
EVENTS
ILS

BY PAUL LINN

CENTRAL IOWA LOW-ACHIEVER
MATHEMATICS PROJECT (CILAMP)
1164 - 26th Street
Des Moines, Iowa 50311

TO THE TEACHER

USING THE PROGRAM

The First Probability Program uses the Crowderian style of programed learning which has proven to be of high interest to most readers. This material is not so piecemeal as it is in regular programing. The reader is challenged by the questions and his answers determine what information he will receive. If he understands the material, he quickly completes the program. If he has difficulties, he is given further explanation and assistance so that he will take longer to complete the program.

Probably, the most common use for this program is for review. (If you are using the CILAMP Probability unit, this program is used to review or introduce the first two chapters.) It contains most of the fundamental ideas of probability suggested for junior high school students. Students can use the program independently or in small groups. Use of this type of material in small homogenious groups has proven to be quite effective.

The program may also serve as an excellent quick but clear overview to introduce the initial study of probability to a class. It is suggested that the presentation be to the whole class using transparencies on the overhead projector.

Students will need a pencil to work the program. They should write lightly and erase their work after they have completed the program. Using scrap paper to work problems may be a better solution in some cases.

SOURCES OF BASIC MATERIALS
FOR TEACHING PROBABILITY TO LOW ACHIEVERS

A. The CILAMP Probability unit is, of course, one of the basic teaching materials that this program can introduce or review (Chapters I and II). A second program is planned to be used with Chapters III-VI.

B. Unit 5: Arrangements and Selections of the National Council of Teachers of Mathematics series Experiences in Mathematical Discovery This fifty-page booklet also discusses Pascal's triangle and tree diagrams which are not covered in the Program. The Program should still serve as a good review for the main ideas of this booklet developed especially for low achievers.

C. Events and Chance is a booklet for low achievers written by Title II Mathematics Project (Building S-503, 3323 Belvedere Road) West Palm Beach, Florida (33401).
The First Probability Program serves as an excellent introduction or review for this unit. This unit may be followed by the West Palm Beach unit, Arrangements and Combinations.

D. Mathematics in Action: The Role of Mathematics in Life Insurance, is a short well written and appealing booklet sent free by the Educational Division of the Institute of Life Insurance (277 Park Avenue, New York, New York 10017). This booklet may serve as a supplement or the basic portion of a teacher-made unit on probability. The First Probability Program serves best as a review since the unit itself is rather brief.
A more detailed treatment using set notation can be found in the Institute of Life Insurance's Sets, Probability and Statistics.

E. Mathematics We Need, J 2, Chapter 7 of the Ginn and Company Basic Series presents materials suitable to mixed classes containing low achievers. The Program can be used to review this section before the more advanced students go on to the enrichment section.

FIRST PROBABILITY PROGRAM

You are not to read this like a book by starting at the beginning and reading each page all the way through. Instead, as you read you will find questions and you will then select the answer that you feel is the best. After that answer will be listed the number of a frame or a question. You should turn to that number and continue reading.

Many kinds of problems that we have worked in math have only one answer and are very useful in solving problems we meet every day. However, we sometimes run into problems where we cannot be certain of the answer. A businessman faces the problems of planning production for next year and says that sales will probably exceed $3,000,000. A weatherman says that there is a 20% chance of rain tonight. A sports writer predicts Valley will win the ball game next Friday. Problems like these, where the answer is not certain, are called probability problems.

Probability problems have little to do with most areas in our complex society where answers must be exact.

I agree. (turn to #10)   I disagree. (turn to #2)

#1. You goofed and are not following the directions. Read the instructions again and when you answer a problems follow the path you are directed to take.

#2. If you toss a coin, it can land either "heads up" or "tails up." If we ignore the rare chance that a coin will land on its edge, there are just 2 possibilities which can occur. If a coin is tossed once the possibility of getting heads is 1 out of 2 or 1/2. Sometimes a coin will be heavier on one side or worn so that one side comes up more frequently than another side. Such a coin is said to be not "fair." Bob flips a coin that is "fair" 10 times and it comes up tails 9 times. What is the probability that it will be tails on the next toss?

9/10. (turn to #3)   1/10. (turn to #4)   1/2. (turn to #12)

#3. Go on to #4.

#4. If the coin is "fair" the probability that it will come up tails is always 1/2. If you put three marbles in a cup (2 red ones and 1 green one), what would be the probability of getting a green one if you didn't look when you took one out?

1/3. (turn to #9)   1/2. (turn to #12)   1/4. (turn to #24)

#5. Oh! Oh! You aren't following directions again. Why not? "I don't understand the directions." Read them again and if you still have questions, ask for help. "I was just curious about what I might be missing." Don't be nosey about all these questions and follow the pattern designed for you.

#6. You must be kidding. It isn't fair not to be honest. Turn to #2 and follow the directions.

#7. You are correct. What relationship do you see between the denominator (bottom number in the fraction) of the probability fraction and the number of possible outcomes for the problem?

None. (turn to #14)   They are the same. (turn to #11)

#8. Wrong. The answer depends upon the number of each color of marbles. (turn to #13)

#9. Correct. (turn to #12)

#10. Let's look at some of the uses of probability in our society. They are more important then you thought.

#10 continued--

Insurance---------------------To determine insurance rates.

Major Industries--------------For production control and possible sales.

Medical profession------------In research, treatment, and the prescription of medicines.

Military----------------------Strategies of battles.

Government--------------------Tax programs, spending, and many other areas.

Large-scale farmers-----------To plan and harvest crops.

Educators---------------------To plan facilities, programs, and advise students.

Cities------------------------To plan building programs, civic activities, police and fire protection, and many other areas.

Politicians-------------------To plan election campaigns.

Advertising-------------------To develop effective advertising plans.

Gambling establishments-------Guess why.

Do you feel that probability is important in our lives?

Yes. (turn to #2)   No. (turn to #6)

#11. Correct. What relationship do you see between the numerator (top number in a fraction) of the probability fraction and number of ways the event you are talking about could happen?

None. (turn to #15)   They are the same. (turn to #25)

#12. You are correct. If you had four marbles (3 red and one green) in a cup, what would be the probability of getting a red one without looking as you took one out?

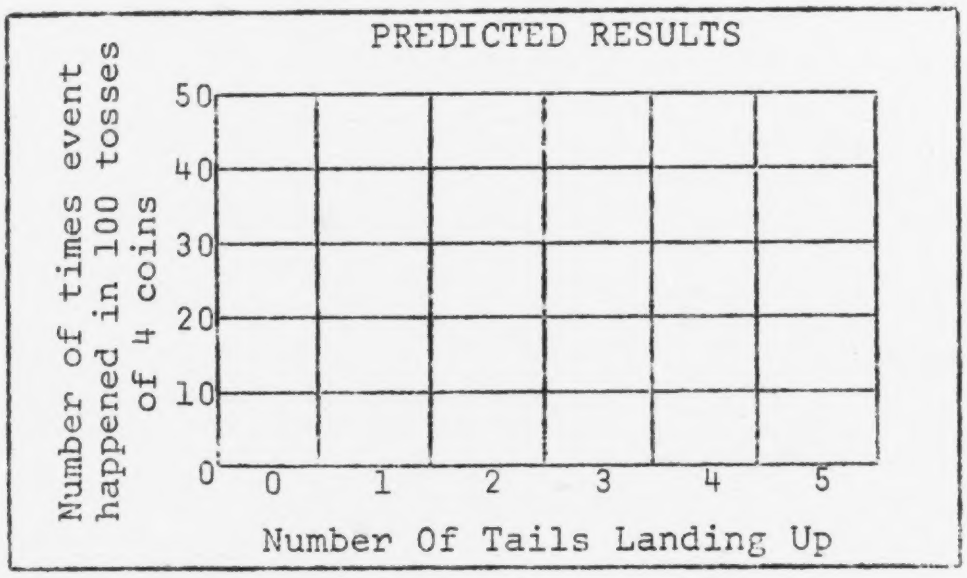
1/3. (turn to 13)   3/4. (turn to 7)   1/2. (turn to 8)

#13. If you had one red and one green marble, the chance of getting a red one would be 1 out of 2 or 1/2. If you had two red ones and one green one, the chance of getting a red one would be two chances out of three or 2/3. (turn to #7)

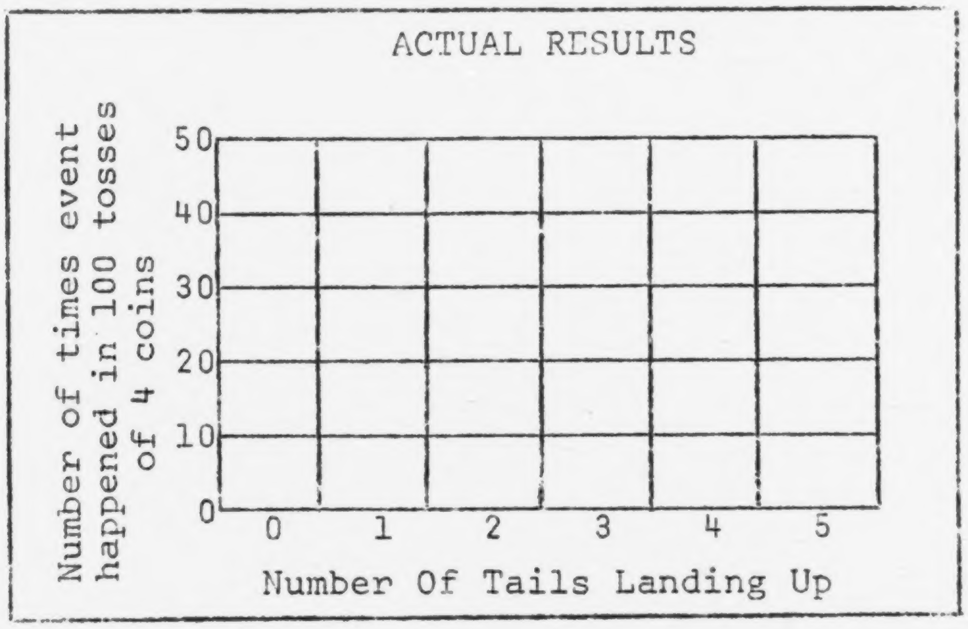
#14. There are six sides on a die. Only one of those sides has two dots on it. The chance of this side landing up is one in six or 1/6. There are 30 students in your class. If they each draw a slip of paper from a hat containing 29 plain slips and 1 slip with a special mark, the probability of your getting the slip with the special mark is 1 out of 30 or 1/30. In each case the denominator is the same as the number of possibilities. (turn to #11)

#15. Two (2) students from your class of 30 may go on a special expense-paid trip. No one wants to decide who the lucky ones will be so you agree to put 28 plain slips of paper in a hat and 2 with a special mark. Each student draws out a slip. You will have 2 chances out of 30 or 2/30 to be one of the winners.
There are 4 aces in a deck of 52 cards. Your probability of drawing out an ace at random would be 4 out of 52 or 4/52. In each case, the numerator is the number of possibilities that you have of drawing what you are lookinf for from the total possibilities. (turn to #25)

#16. Predict how many times 0, 1, 2, 3, 4, and 5 tails will show up if you flip 4 coins at a time and do this for 100 tries. Plot your prediction on the following graph.



Now try it yourself and plot your results.



What can you say about the comparison of actual results and predicted results when you use a small number of tries compared to when you use a large number of tries?

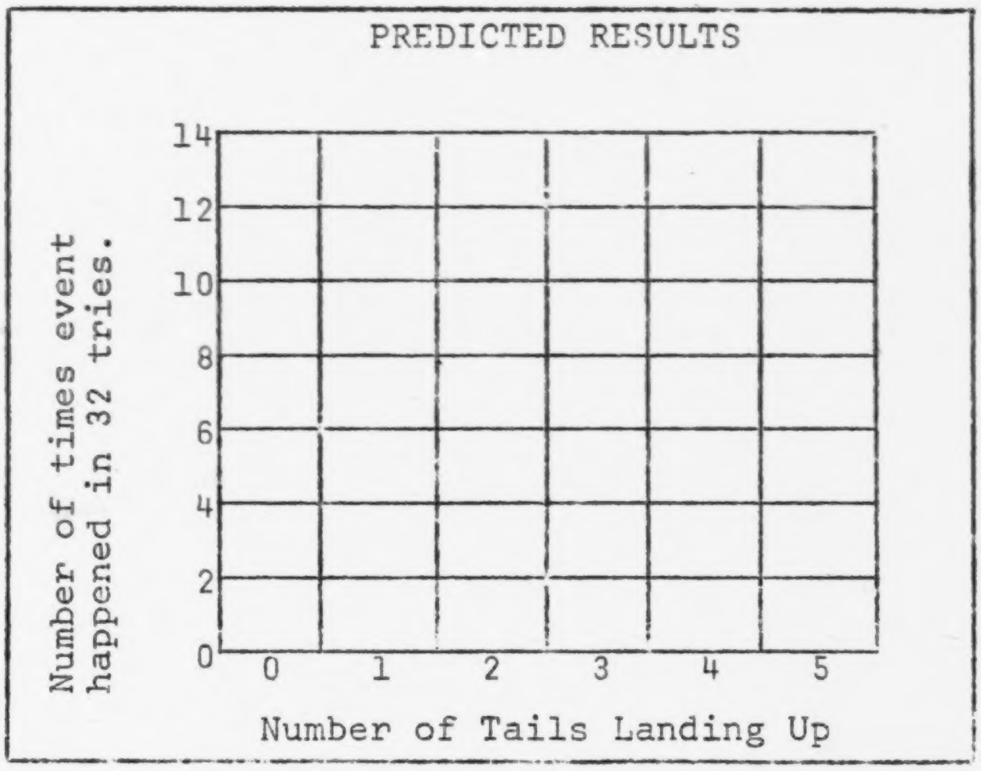
There is no relationship. (turn to #21)

The large number of tries will always come closer to agreeing. (turn to #22)

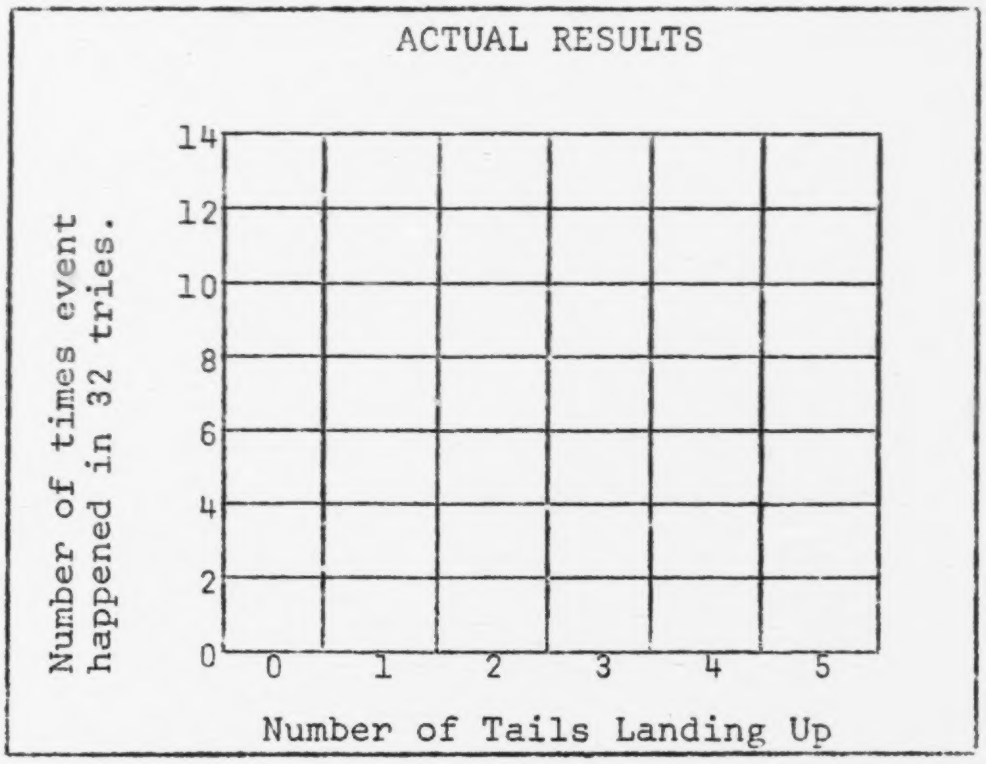
The large number of tries generally will come closer to agreeing. (turn to #20)

I don't think my graph of predicted results is correct so I'm not certain of any relationship. (turn to #28)

#17. Use your predictions from number 23 to graph the results of flipping 4 coins 32 times on the graph below.



Now flip 4 coins 32 times and graph the actual results below.

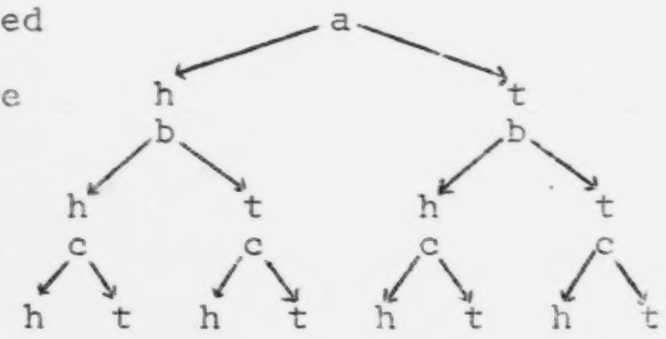


(Go to #26)

#18. Suppose you flip three coins (coin "a," coin "b," coin "c"). What are the possible arrangements? "a" could be heads (represented by h) or it could be tails (represented by t). With "a" as heads there are two ways that coin "b" could land. This would give four possible ways that "a" and "b" could be put together. With each of these ways there are two ways that "c" could land. This would mean that there are how many possible different arrangements of the coins?

8. (turn to #17)   4. (turn to #27)

EXAMPLE



#19. There are 16 possible arrangements of the four coins.

	Four coins: 1	2	3	4
One possible arrangement of 4 coins=	Ⓣ	Ⓣ	Ⓣ	Ⓣ
Another possible arrangement =	Ⓣ	Ⓣ	Ⓣ	Ⓗ
Another arrangement =	Ⓣ	Ⓣ	Ⓗ	Ⓣ
" " =	Ⓣ	Ⓣ	Ⓗ	Ⓗ
" " =	Ⓣ	Ⓗ	Ⓣ	Ⓣ
	Ⓣ	Ⓗ	Ⓣ	Ⓗ
	Ⓣ	Ⓗ	Ⓗ	Ⓣ
	Ⓣ	Ⓗ	Ⓗ	Ⓗ
	Ⓗ	Ⓣ	Ⓣ	Ⓣ
	Ⓗ	Ⓣ	Ⓣ	Ⓗ
	Ⓗ	Ⓣ	Ⓗ	Ⓣ
	Ⓗ	Ⓣ	Ⓗ	Ⓗ
	Ⓗ	Ⓗ	Ⓣ	Ⓣ
	Ⓗ	Ⓗ	Ⓣ	Ⓗ
	Ⓗ	Ⓗ	Ⓗ	Ⓣ
	Ⓗ	Ⓗ	Ⓗ	Ⓗ

There is just as good a possibility for the first one to occur as for any of the others. We could expect to have all tails 1 time out of 16 or 1/16 of the time. 1/16 is called the probability fraction for 4 tails. What is the probability fraction for 3 tails? for 2 tails? for 1 tail? for 0 tails?

(turn to #23)

#20. Very good. You are now finished with this booklet.

#21. You are right that you can't be certain of relationships but, generally, the more times you try the closer your results will come to the ones you predicted.

(turn to #20)

#22. In most cases the larger the number of tries the closer to each other will be the results and predictions but there is always the possibility of a small number agreeing exactly and a large number being way off.

(turn to #20)

#23. The correct answer is as follows:

0 tails = 1/16 = 6 1/4%

1 tail = 4/16 = 1/4 = 25%

2 tails = 6/16 = 3/8 = 37 1/2%

3 tails = 4/16 = 1/4 = 25%

4 tails = 1/16 = 6 1/4%

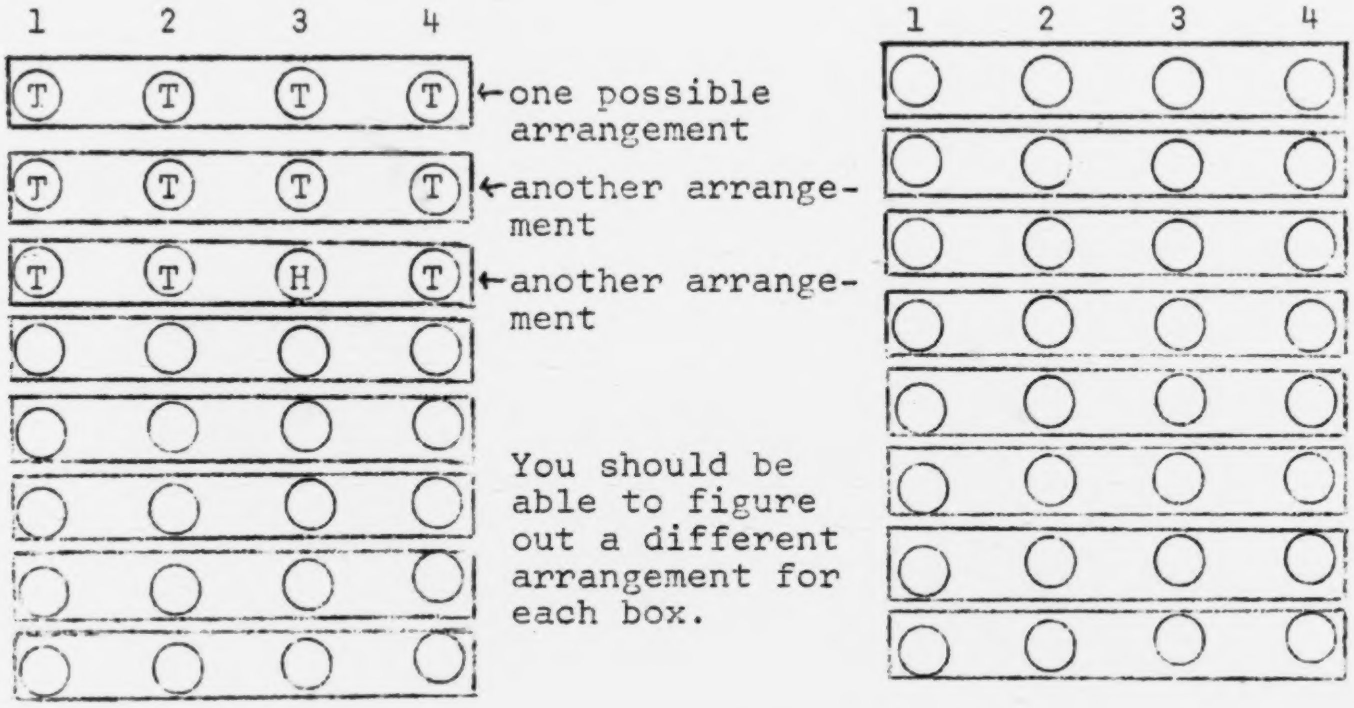
Do you see how the above answers were determined?

Yes. (turn to #17) No. (turn to #18)

#24. One of the three marbles will give the results you are looking for so the probability of getting the 1 green one would be 1/3. The probability of getting a red one would be 2/3.

(turn to #12)

#25. You have discovered the rules of probability. The denominator of the probability fraction is the same as the number of possible arrangements. The numerator is the same as the number of these arrangements that give the results that you are looking for. Now you are ready to try your rules. Take four coins and put them in a box to shake. Calculate the probability that 0 tails will show up. Calculate the probability that 1 tail will show up; that 2 tails will show up; that 3 tails will show up; that 4 tails will show up. Remember that there is one way for all tails to show up but 4 different ways for 3 tails to show up. Put a "T" on the circles for coins that land tails up in the circles below and an "H" on the circles for coins that land heads up in the circles below. Work out all the possible arrangements. Each row of 4 coins represents a possible arrangement. Check your answer by turning to #19.



←one possible arrangement

←another arrangement

←another arrangement

You should be able to figure out a different arrangement for each box.

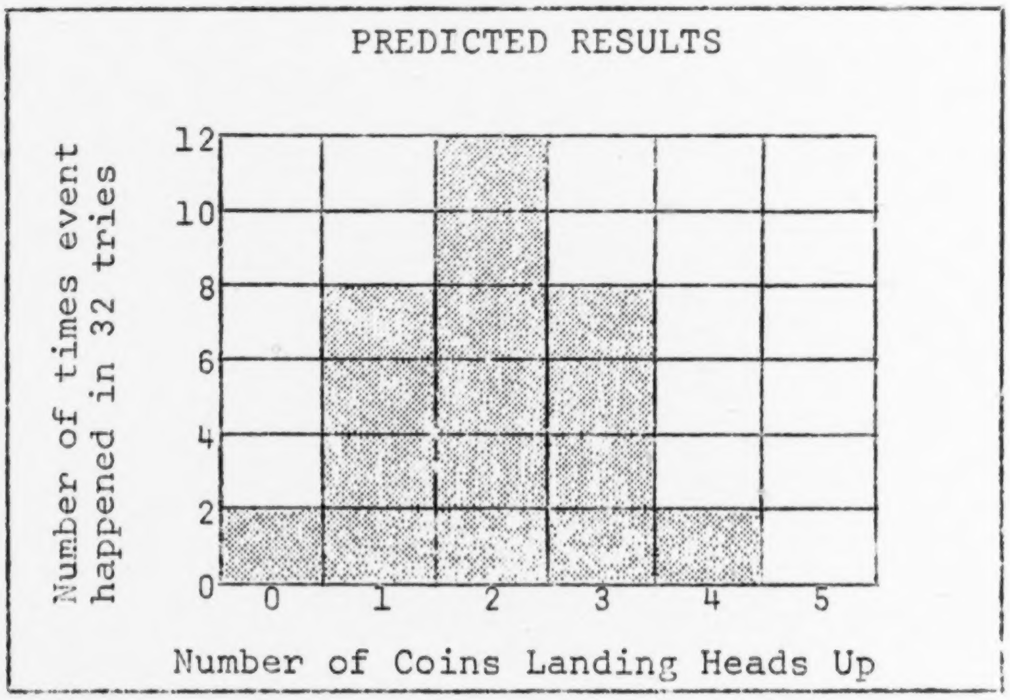
#26. Your predicted results should have looked like this:

1/16 of the time you predicted 0 tails.

1/16 of 32 would be 2. (1/16 x 32 = 2)

You predicted 1/4 of the time 1 head

would come up. (1/4 of 32 is 8)



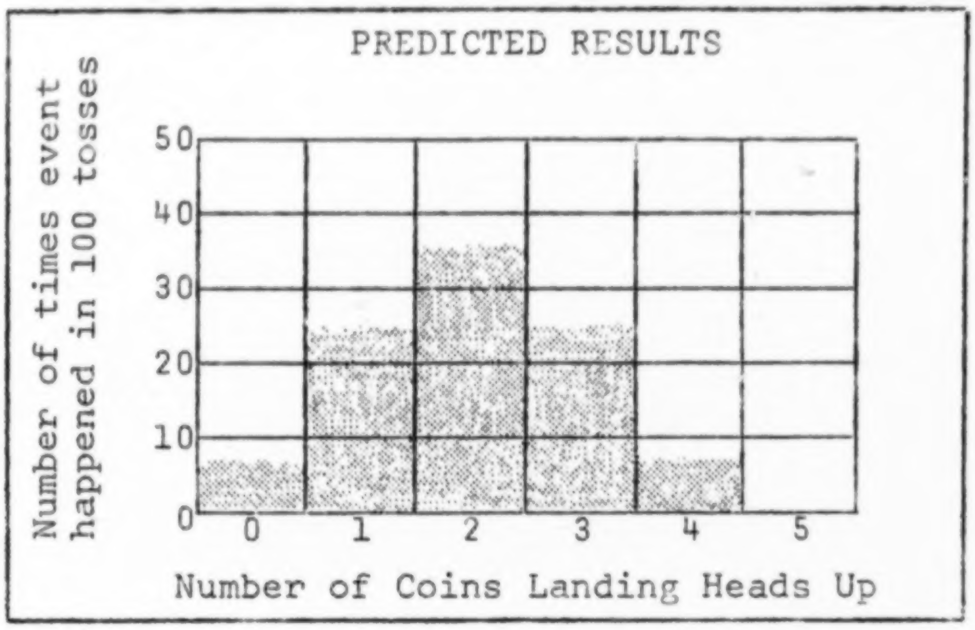
(turn to #16)

#27. There are eight ways that 3 different coins could land.

1	2	3	
Ⓗ	Ⓗ	Ⓗ	1 way
Ⓗ	Ⓗ	Ⓣ	2 ways
Ⓗ	Ⓣ	Ⓗ	3
Ⓗ	Ⓣ	Ⓣ	4
Ⓣ	Ⓗ	Ⓗ	5
Ⓣ	Ⓗ	Ⓣ	6
Ⓣ	Ⓣ	Ⓗ	7
Ⓣ	Ⓣ	Ⓣ	8

(turn to #17)

#28. Your predicted results should have looked like this.



(turn to #22)